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Equivalent load model of induction machines connected to a common bus

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EQUIVALENT LOAD MODEL OF INDUCTION MACHINES CONNECTED TO A COMMON BUS

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I. LOAD MODELS

A. Introduction

Power system analysts require mathematical models to represent the various components in the system. The accuracy of the results of any analysis depends on how good the models are. Fairly detailed representations have been developed for synchronous machines and transmission lines. However, loads are extremely difficult to model, especially composite loads. These loads are not homogeneous and, therefore, are difficult to represent by a simple model or mathematical formula. Furthermore, as in the case of induction motor loads, the characteristics of individual loads could vary widely. To complicate matters further, some loads, like arc furnaces, are stochastic.

Since load models are indispensable in power system analyses, various types have been proposed and used. These range from representing loads in electrically remote areas of the system under study as negative generation to very detailed models of individual load components within the study area. The type of studies, the accuracy desired, and the availability of load models dictate which models are used. Models suitable for load flows may not be accurate enough for transient stability studies or for analyzing the performance and interactions of various loads. The choice of a particular model depends on the complexity of the system being analyzed and the desired accuracy of the results.

B. The Importance of Load Models in the Analysis of Electric Power Systems

The apparent power drawn by loads in power systems varies with both the voltage and frequency. During electrical power system disturbances, these variations and the load dynamics determine the amount of real and reactive power consumed by the loads. However, most of these changes are very complex and not easily modeled. Therefore, loads have been frequently modeled in power system analyses by constant impedances (constant Z), constant currents (constant I), or constant apparent power (constant MVA); but these models do not adequately describe the load behavior in all cases.

For constant impedance loads, the apparent power consumed varies as the square of the voltage. For constant current loads, the apparent power is directly proportional to the voltage, but for constant MVA loads it is independent of the voltage. Some loads cannot be accurately described by any single one of these representations and combinations of all three are sometimes used [25] in building the model of the load. In other instances, simple mathematical equations, that are usually polynomials or exponentials of voltage and frequency [11, 19, 2, 6], are used to relate these variables to the real and reactive power consumed by the loads.

Different load models are known to give different results for transient stability studies. In some cases [15, 25, 19, 34, 43], one load model may give results which predict that a power system is stable following a disturbance, while another model would predict instability for the same disturbance. Conflicts of this nature require that the behavior of various loads during and following power system disturbances be known and accurately modeled in order that conclusive deductions could be made. This knowledge is very essential especially when the results depend on the load model.

A study by Gentile et al. [14] shows that load representation sometimes affects the outcome of a transient stability study more than the response ratio of generator excitation systems. In this study, it was observed that changing the load model from constant impedance to constant current changed the results of the transient stability study more significantly than the changes observed when the response ratios of the exciters were changed from 0.5 to 2.0. This is significant [14] because the decision on response ratio for exciters could involve large equipment costs, up to a million dollars in some cases.

Another study [14] shows a case in which the transient stability limit of the power transfer between two power pools, the New York and New England power pools, was significantly limited by uncertainties in load characteristics. In this interconnection, the thermal limit of the power transfer between the two systems was known to be between 1400 MW and 1500 MW. Despite this, the transient stability limit was set at 1200 MW because the load model (constant impedance in this instance) that appeared to give the most conservative results was chosen in setting the power interchange limit. Lack of an adequate model for the load prevented designers from taking full advantage of the interchange capabilities.

Since more than 60% of the load in electrical power systems is driven by induction motors [42], the dynamics of these motors and their mechanical loads would account for the greater part of the dynamic power requirements of the load. In some analyses, it is important to model this dynamic behavior. When load dynamics were modeled in a transient stability study done on a two-generator system [16], the results showed that the system was less stable than when only static load representations (constant Z, constant I), were used. This was the case for remote generation. The opposite of these results was obtained for local generation.

C. Load Model Literature Review

Much work has been done to determine the steady state characteristics of loads. Kent et al. [25] have given a method that uses voltage and power measurements at a load bus to determine the decomposition of the load into constant Z, constant I, and constant MVA components. The percentage decomposition is shown to depend on the the type of area the bus feeds (for example, residential, commercial and industrial area), and the types of loads connected to it. Heating loads usually behave as constant impedances and motors behave more like constant MVA loads.

In their experiments, Kent et al. caused voltage variations at various load buses and measured the steady state real and reactive power requirements and the voltage. The results are then used in calculating the load decomposition into constant Z, constant I, and constant MVA components. This decomposition is then used in estimating the steady state power requirements at any other voltage level.

In practice, it is difficult to determine the load characteristics at all load buses by creating disturbances and carrying out measurements. Furthermore, it is desirable to predict load response to more drastic changes in bus voltage than is feasible or advisable when carrying out experiments in an operating system. Bus voltages vary between 0 and 120% of the normal value in real systems [13]. However, in experiments the variations are usually limited [19] to only ±7.5% of the nominal value when transformer taps are used to cause the voltage changes. There is, therefore, the need to obtain the load response by different means.

Other researchers like Chen [10, 11, 12] and Berg [6] have approached the problem of modeling loads differently. They build the response of a composite load as the sum of the responses of the various load components. The behavior of most load components when operated at voltages or frequencies other than their rated values have been measured and the data from these measurements are used in the building process.

Simple statistical methods are used to estimate the number of individual load components connected to any bus in the system. Load inventory and load utilization data are used to estimate the total load connected to any bus. The load inventory data, which can be obtained from the manufacturers or other sources [16], gives the number of components of a kind (e.g., refrigerators, electric ranges) connected to the system, and the load utilization data gives the fraction of the connected load in use at any time.

The results of measurements to determine the voltage and frequency characteristics of power system load components have been reported in the works of several authors [10, 11, 2, 3, 6]. These results are usually expressed as equations which relate the real and reactive values of power to polynomial or exponential functions of voltage and frequency. Sometimes [10, 11] the deviations of voltage and frequency from their nominal values are used in the equations for more accurate results.

Even though this method describes static loads accurately, it gives poor results for dynamic loads. Electric motors and their mechanical loads contribute most to the load dynamics. The parameters of the motor, its rating and operating conditions, and the nature of the mechanical load, all affect the response of a motor following a disturbance.

Since more than 60% of power system loads are driven by induction machines, much attention was given to modeling induction machine loads during system disturbances. Early in the art, Brereton and colleagues [8] developed various simplified models for induction machines for use in transient stability studies and showed the limitations of some of the simplified models.

Other researchers who studied the behavior of induction machine loads in power systems include Magginnis and Schultz [32], Ramsden, Zorbas, and Booth [35], Mauricio and Semlyen [34], Gevay and Schippel [18], Kalsi and Adkins [24], Mahmoud, Harley and Calabrese [33], Iliceto and Capasso [21], and Ueda and Takata [43]. These researchers studied different aspects of machine loads, especially their effects on the bus and network voltages during starting and on the transient stability (of both the machine and system) following disturbances. Many other researchers have worked on different aspects of the effect of induction machine load disturbances on power systems and vice versa, but all of them have not be mentioned here.

The behavior of single induction machines in power systems has been described by equations developed by several researchers. Equations derived by Stanley [41], Krause [26], and Krause and Thomas [29] are widely used for studying the steady state and transient behavior of induction machines. Another group of authors, which include Jordan [23], Robertson and Hebbar [36], Lipo, Krause and Jordan [31], and Krause and Hake [27], have also developed methods that analyze the transient behavior of inverter-fed motors.

D. Transient Response of Induction Machines

There are three main components of the transient response of induction machines. These are the electrical transients of the stator circuit, the electrical transients of the rotor circuit, and the mechanical transients of the rotor and its connected load. The accuracy obtained from any simulation depends on the number of these transients taken into account. When all three components are represented in a model, such a model is said to be full order. Neglecting one or more components results in a reduced order model. Brereton and colleagues [8], Berg and Subramaniam [7], Krause and Murdock [28], Sastry and Burridge [38], Skvarenina and Krause [40] and Krause et al. [30] have all explored the use of reduced order models and have given the limitations of some of the models.

Whereas modeling all three transient phenomena results in the highest degree of accuracy, it involves the solution of more equations and takes more computing time. Various researchers including Berg and Subramaniam [7] and Chen [10, 11] studied the responses of the various models and compared them with experimentally measured results. They have shown that the full order model (of fifth order for single-cage induction machines) most accurately tracks the variations in current and power during transients. A third order model only gives the average response of the stator variables but neglects the high frequency variations in the stator current and power which usually occur following a disturbance [10, 11]. The first order model, in which only the mechanical dynamics of the rotor are simulated, is accurate [10, 11, 7] only after the transients have died down.

During some severe disturbances, the frequency of the electric supply system deviates from its nominal value. Not much [12, 9] research has been done to determine the effect of such deviations on induction machines. Akhtar [3] reported a method by which the steady state power drawn by a group of motors could be calculated or estimated when the frequency deviates from the nominal value.

E. Single Unit Equivalents of Two or More Induction Machines

There are many induction machine loads in power systems and it is impractical to represent all these machines in most power system analyses. Several attempts [3, 7, 1, 37, 21, 5] have, therefore, been made to represent the composite effects of the dynamics of two or more induction machines and their mechanical loads by an equivalent machine and load. Akhtar [3] proposed a method that deals only with the effect of frequency variations on groups of induction motor loads. The method assumes the response of the motors to frequency change is exponential. The method also gives an equivalent motor which represents the group of induction motors. The response of this equivalent motor to frequency changes is also exponential and not the dynamic (oscillatory and damped average) response that is usually obtained from measurements.

Another method was formulated by Abdel-Hakim and Berg [1] for calculating the equivalent of a number of machines fed from a common bus. In this method, all the machines are represented by their equivalent circuits. These circuits are connected in parallel and are then reduced to a single equivalent circuit that represents the single unit equivalent. The concept of conservation of mechanical output power is used in finding the torque speed characteristics of an equivalent mechanical load. This method has been used by Berg and Subramaniam [7] and Roohparvar, Mahmoud and Hanania [37] to study various behaviors of induction machine loads during transients and under motor starting conditions. The equivalent obtained by this method is of first order only and does not adequately model the dynamics of the load during the period that follows a system disturbance [7, 10, 11], when the electrical transients are significant. It is, however, accurate in the steady state.

Chen [10, 11, 12] has also proposed another model that uses two exponential functions with the same time constant to describe the variations of real and reactive power drawn by induction motors following a disturbance. The equivalent time constant of two or more motors is found as a weighted sum of their individual time constants, the weighting factors being the steady state real powers drawn by the individual motors. Other researchers, namely Gentile, Ihara, Murdoch and Simons [14, 16], did experiments to test this method and found that it does not adequately model load dynamics. The model is reported to predict time constants that are five to ten times lower than measured values.

Another method of obtaining single unit equivalents was developed by Iliceto and Capasso [21]. They used a third order model (which simulates the rotor electrical and mechanical transients of the induction machines) in deriving the equivalent. The open circuit time constant, T'do, and the inertia constant, H, of the rotor and load are important parameters which greatly affect the dynamic response of any motor. The authors proposed that the T'do of the equivalent machine may be calculated as the weighted average of the T'do of the individual motors. The weighting factors are the ratios of the motor power ratings to the total power rating of all the motors to be grouped together. The same weighting is used in obtaining the inertia constant, H, the resistances and reactances of the equivalent machine. Iliceto and Capasso also found that not all motors could be combined into one equivalent, but only those satisfying the condition $H > (1/2)\ T'_{do}$.

Ihara and Baheti [20] developed a dynamic load model that represents a large number of small and medium induction motors by a number of 'basis motors'. The method divides motors into classes based on their ratings. From these divisions, a representative motor is chosen to represent the 'typical' motor in each class. This is called the 'basis motor'. Using 'basis motors' instead of the actual motors in the system makes data preparation very minimal.

The inertial dynamics of the 'basis motors' are formulated in an admittance form which yields a simpler mathematical method of aggregation. A number of assumptions and simplificatons are made in deriving the model. The two most important ones are:

1. Neglecting both stator and rotor flux transients.
2. Restricting voltage dips.

The restriction on voltage dips requires it to be composed of a voltage depression of short duration (a fault dip) followed by a slow and limited swing (a swing dip). Although necessary to prevent motors from stalling during the dip, Ihara and Baheti [20] pointed out that these restrictions might greatly reduce the practicality of the model.

F. Summary

There are some motivations in searching for equivalents to be used in power system analyses. Using an equivalent motor and mechanical load to represent all or some of the motor loads connected in the power system would greatly reduce the number of mathematical equations that are required to describe all the motor loads. In addition, a good equivalent induction motor and mechanical load would give an accurate load dynamic behavior that would represent the dynamics of the many motor loads connected to the system bus. This equivalent would also take into account the variations in the real and reactive power consumed by the load due to both voltage and frequency changes. The limitations of the equivalents discussed so far render them inadequate for use in solving every problem in power system analyses.

G. Research Scope and Objectives

Much work has been done in determining the voltage and frequency dependence of the real and reactive power drawn by many nondynamic load components. An adequate model for the dynamic portion of the load is needed for some power system analyses like stability studies. Even though a number of models have been proposed, none has been found to accurately describe the load behavior during transients. This research is directed towards:

1. Developing equations for a single unit equivalent of squirrel cage induction machines (fed from a common bus) using their transient equations in the 0-d-q axes.

2. Testing the derived equivalent machine against the full representation of all the machines.
3. Determining criteria for grouping different induction machines into single units with minimum error.
4. Comparing the resulting dynamic load model with constant current and constant impedance load models in a transient stability study.
5. Suggesting how the method of this work could be employed in modeling induction machine loads in practical power systems.

II. AN EQUIVALENT MODEL OF N INDUCTION MACHINES

A. Introduction

An induction machine and its performance can be analyzed by a set of electrical and mechanical equations derived for the machine. The electrical equations are either flux linkage or voltage equations of the stator and the rotor circuits. The mechanical equations describe the inertial dynamics of the rotor and any attached mechanical load. Both the electrical and the mechanical equations are needed to fully describe the machine performance.

The electrical equations may be written in terms of the a-b-c phase variables and then transformed by Park's transformation into the known orthogonal 0-d-q axes. This transformation simplifies the solution of the equations for symmetrical machines. Inductances that vary with the angular position of the rotor in the a-b-c quantities become constant after the 0-d-q transformation. Details of this transformation from the a-b-c variables to the 0-d-q variables are summarized in Appendix A.

The development of the equations in Appendix A is clearly oriented towards the analysis of single induction machines since it involves mutual couplings that are present between two magnetic circuits in the same machine. For analyzing a single machine connected to a load bus in a power system, these equations are adequate. However, there is more than one induction machine at most load buses in an electric power system. It is therefore desirable to derive a new set of equations which would describe a whole group of induction machines connected to a common bus. This dissertation is focused on developing such a model.

In practice, induction machines are fed from load buses through feeder cables. In some cases, these feeders are short enough that their impedances can be neglected in any analysis and the voltage at the bus assumed to be the same as the voltage at the machine terminals. But in other cases, the impedances are significant and can not be neglected without causing significant error. A large feeder impedance reduces the voltage at the terminals of the machine thus affecting its performance. For accurate analysis, it is desirable to include these feeders in any studies done on the machines and the system.

In the development that follows, it is assumed that a group of induction machines are fed from a common bus through separate feeders. In order to derive an equivalent machine to represent all these machines, the parameters of each feeder and the voltage drop across it are incorporated into the equations of the machine it feeds. The resulting equations are then used to develop the equations of an induction machine which is equivalent to the whole group of induction machines. This equivalent machine is connected to the original common bus. It has its own parameters and inertia and draws current and power equal to the sum of the currents and power drawn by the individual machines. It also drives an equivalent mechanical load and delivers the same power to the mechanical load as the total power delivered by the individual machines.

B. Incorporating Feeder Parameters into the Induction Machine Equations.

As shown in Appendix A, the voltage equations that describe the behavior of a symmetrical three phase induction machine are given by Eq. 2.1.

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qr} \\ v_{dr} \end{bmatrix} = \begin{bmatrix} R_s + L_{ss}p & -\omega L_{ss} & Mp & -\omega M \\ \omega L_{ss} & R_s + L_{ss}p & \omega M & Mp \\ Mp & -(\omega-\omega_r)M & R_r + L_{rr}p & -(\omega-\omega_r) L_{rr} \\ (\omega-\omega_r)M & Mp & (\omega-\omega_r)L_{rr} & R_r + L_{rr}p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \tag{2.1}$$

where

v_{qs} = q-axis stator voltage

v_{ds} = d-axis stator voltage

v_{qr} = q-axis rotor voltage

v_{dr} = d-axis rotor voltage

R_s = resistance of stator circuit

L_{ss} = inductance of stator circuit

p = d/dt

ω = speed of reference frame

ω_r = speed of rotor

L_{rr} = inductance of rotor circuit

M = mutual inductance between rotor and stator circuits

For singly fed induction machines, $v_{dr} = v_{qr} = 0$. In Eq. 2.1, the voltages v_{ds} and v_{qs} are the d-axis and the q-axis values of the transformed stator terminal voltage V_T shown in the single line diagram of Fig. 2.1. In the figure, the induction machine is fed from the bus B through a balanced three phase feeder having resistance R_F and inductance L_F.

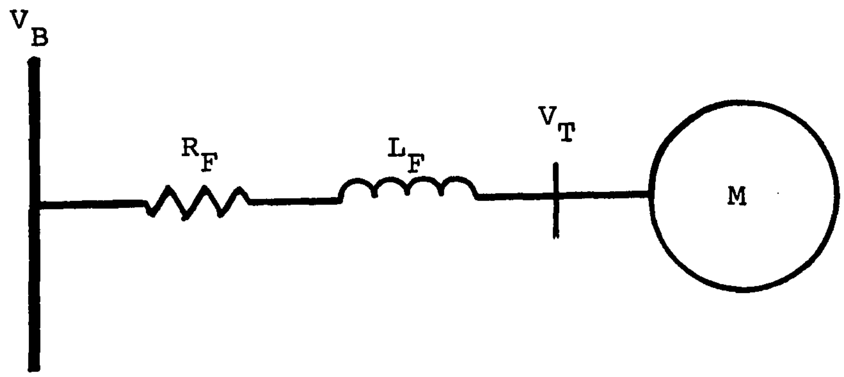

Fig. 2.1. Single line diagram of an induction machine and its feeder

From Fig. 2.1, the voltages at bus B are given by

$$[V_B]_{abc} = [V_T]_{abc} + [V_F]_{abc} \qquad (2.2a)$$

$$= [V_T]_{abc} + [R_F]\,[i]_{abc} + [L_F]\,p[i]_{abc} \qquad (2.2b)$$

where

$$[R_F] = \begin{bmatrix} R_f & 0 & 0 \\ 0 & R_f & 0 \\ 0 & 0 & R_f \end{bmatrix} \qquad (2.2c)$$

$$[L_F] = \begin{bmatrix} L_f & L_m & L_m \\ L_m & L_f & L_m \\ L_m & L_m & L_f \end{bmatrix} \qquad (2.2d)$$

$$[i]_{abc} = [i_a \quad i_b \quad i_c]^T \qquad (2.2e)$$

$[V_B]_{abc}$ = the phase voltage at bus B

$[V_T]_{abc}$ = the terminal voltage of the induction machine

$[V_F]_{abc}$ = the voltage drop across the feeder

$p = d/dt$

Since it is easier to analyze the induction machine using the 0-d-q axes variables, Eq. 2.2a will be transformed from the phase variables into the 0-d-q. Equation 2.3a illustrates the transformation.

$$[P]\,[V_B]_{abc} = [P]\,[V_T]_{abc} + [P]\,[R_F]\,[i]_{abc} + [P]\,[L_F]\,p[i]_{abc} \qquad (2.3a)$$

where

$$[P] = \sqrt{2/3}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3) \\ \sin\theta & \sin(\theta-2\pi/3) & \sin(\theta+2\pi/3) \end{bmatrix} \quad (2.3b)$$

[P] is the modified Park's transformation matrix and

$$[P]\,[V_B]_{abc} = [V_B]_{0dq} \quad (2.3c)$$

$$[P]\,[i]_{abc} = [i]_{0dq} \quad (2.3d)$$

The term $[P]\,[V_T]$ of Eq. 2.3a are the zero, d-axis and q-axis transformed values of the stator terminal voltage V_T which are equal to the stator voltages given in Eq. 2.1 where the zero sequence quantities are zero for a balanced system. The other terms in Eq. 2.3a are as follows:

$$[P]\,[R_F]\,[i]_{abc} = [P]\,[R_F]\,[P]^{-1}\,[i]_{0dq} = [R_F]\,[i]_{0dq} \quad (2.4a)$$

The last term in Eq. 2.3a is given by

$$[P]\,[L_F]\,p[i]_{abc} = [P]\,[L_F]\,p\{[P]^{-1}\,[i]_{0dq}\} \quad (2.4b)$$

$$= [P]\,[L_F]\,[P]^{-1}\,[P]\,\{p([P]^{-1})\,[i]_{0dq} + [P]^{-1}\,p[i]_{0dq}\} \quad (2.4c)$$

$$= [L_{0dq}]\,[P]\,p([P]^{-1})\,[i]_{0dq} + [L_{0dq}]\,p[i]_{0dq} \quad (2.4d)$$

where

$$[L_{0dq}] = [P]\,[L_F]\,[P]^{-1} = \begin{bmatrix} L_f+2L_m & 0 & 0 \\ 0 & L_f-L_m & 0 \\ 0 & 0 & L_f-L_m \end{bmatrix} \quad (2.4e)$$

and

$$[P]\ p[P]^{-1}\ [i]_{0dq} = \omega \begin{bmatrix} 0 \\ i_{qs} \\ -i_{ds} \end{bmatrix} \qquad (2.4f)$$

Therefore,

$$[P]\ [L_F]\ p[i]_{abc} = [L_{0dq}]\ p[i]_{0dq} + \begin{bmatrix} 0 \\ \omega(L_f-L_m)i_{qs} \\ -\omega(L_f-L_m)i_{ds} \end{bmatrix} \qquad (2.5)$$

Substituting Eqs. 2.1, 2.4a and 2.5 in Eq. 2.3a gives

$$[V_B]_{0dq} = [V_T]_{0dq} + [R_F]\ [i]_{0dq} + [L_{0dq}]\ p[i]_{0dq} + \begin{bmatrix} 0 \\ \omega(L_f-L_m)i_{qs} \\ -\omega(L_f-L_m)i_{ds} \end{bmatrix} \qquad (2.6)$$

For a balanced system, the zero sequence current is zero. Expanding Eq. 2.6 and neglecting the zero sequence gives Eqs. 2.7.

$$[V_B]_d = (\omega L_{ss})i_{qs} + (R_s + L_{ss}p)i_{ds} + (\omega M)i_{qr} + (Mp)i_{dr} + \omega(L_f-L_m)i_{qs} + (R_f + (L_f-L_m)p)i_{ds} \qquad (2.7a)$$

$$[V_B]_q = (R_s + L_{ss}p)i_{qs} - (\omega L_{ss})i_{ds} + (Mp)i_{qr} - (\omega M)i_{dr} + (R_f + (L_f-L_m)p)i_{qs} - \omega(L_f-L_m)i_{ds} \qquad (2.7b)$$

Equations 2.7a and 2.7b are the basic voltage equations for the induction machine and its feeder. By comparison, it is clear that Eqs. 2.7a and 2.7b are similar to the stator voltage equations of Eq. 2.1. Equations 2.7 can be interpreted as induction machine equations in which the resistance R_f of the feeder is added to the stator resistance R_s and the inductance $(L_f - L_m)$ of the feeder is added to the stator inductance L_{ss}. The voltage on the resulting circuit is, therefore, the bus voltage V_B.

Incorporating the feeder parameters into the machine gives the common reference bus voltage needed for deriving the equations of an equivalent machine. It is, therefore, possible to group induction machines that are directly connected to a bus and those connected to the same bus through very long feeders into one equivalent machine at the common bus.

C. Deriving a Single Set of Equations for N Induction Machines Connected to a Common Bus

As is shown in Appendix A, the differential equations that describe a single induction machine may be written as

$$p[i] = [D]\,[V] + [E]\,[i] \tag{2.8a}$$

$$p\omega_r = (T_e - T_m)/2H \tag{2.8b}$$

In Section B, it was shown that the voltage drop equations of each feeder could be incorporated into the machine equations. The resulting equations retain the same form as Eq. 2.8a. However, the quantities in Eq. 2.8a are modified to reflect the incorporation of the series impedance of the feeder. The modified quantities in Eqs. 2.8a and 2.8b then become

$$[i] = [i_{qs} \quad i_{qr} \quad i_{ds} \quad i_{dr}]^T \tag{2.8c}$$

$$[V] = [v_{qB} \quad v_{qr} \quad v_{dB} \quad v_{dr}]^T \tag{2.8d}$$

where the subscript B in Eq. 2.8d refers to the bus behind the feeder in Fig. 2.1 and

[E] = a matrix of machine and feeder parameters, the speed of the reference frame and the speed of the machine rotor, given by Eq 2.8e

[D] = A matrix of machine and feeder parameters given by Eq 2.8i

ω_r = angular speed of induction machine rotor

T_e = electrical torque developed by the machine

T_m = mechanical torque on the rotor

H = total inertia constant of the rotor and the attached mechanical load

The equations of single induction machines and their feeders can be used to find the parameters of an equivalent machine that would represent a group of machines connected to a common bus. Consider the circuit of Fig. 2.2 which is a single line diagram of a group of N induction machines connected to a common load bus. In this diagram, each

$$[E] = \Delta \begin{bmatrix} L_{rr}R_s' & -MRr & -\omega L_{ss}'L_{rr} + (\omega-\omega_r)M^2 & -\omega_r L_{rr} M \\ -MR_s' & L_{ss}'Rr & \omega_r L_{ss}'M & \omega M^2 - (\omega-\omega_r)L_{ss}'L_{rr} \\ \omega L_{ss}'L_{rr} - (\omega-\omega_r)M^2 & \omega_r M L_{rr} & L_{rr}R_s' & -MRr \\ -\omega_r M L_{ss}' & -\omega M^2 + (\omega-\omega_r)L_{ss}'L_{rr} & -MR_s' & L_{ss}'Rr \end{bmatrix} \quad (2.8e)$$

$$\Delta = 1/(M^2 - L_{ss}'L_{rr}) \quad (2.8f)$$

$$L_{ss}' = L_{ss} + (L_f - L_m) \quad (2.8g)$$

$$R_s' = R_s + R_f \quad (2.8h)$$

$$[D] = \Delta \begin{bmatrix} -L_{rr} & M & 0 & 0 \\ M & -L_{ss}' & 0 & 0 \\ 0 & 0 & -L_{rr} & M \\ 0 & 0 & M & -L_{ss}' \end{bmatrix} \quad (2.8i)$$

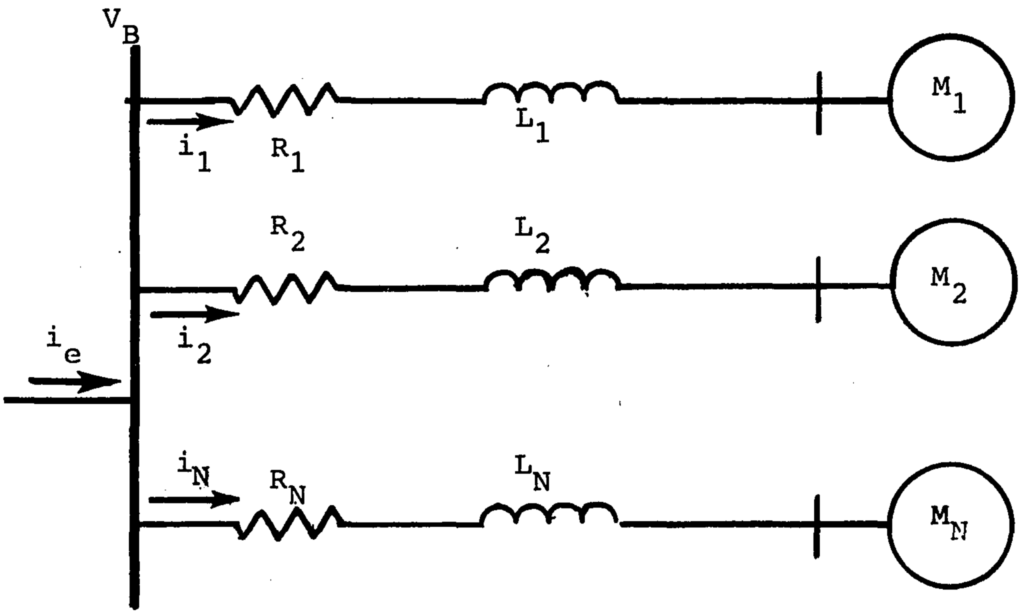

Fig. 2.2. Single line diagram of N induction machines connected to a common bus through their own feeders.

machine is connected through its own feeder to the load bus. The total current at the common bus in Fig. 2.2 is the sum of the individual machine currents. Therefore, the variations in current at the common bus is the sum of the variations in the individual machine currents. Equations 2.9a and 2.9b illustrate these.

$$i_e = \Sigma \, i_k \tag{2.9a}$$

where

$$\Sigma = \sum_{k=1}^{k=N}$$

i_k = current drawn by the k-th induction machine

i_e = total current drawn by all induction machines

$$p(i_e) = p(\Sigma\ i_k) = \Sigma\ p(i_k) \tag{2.9b}$$

The sum of the changes in the currents shown in Eq. 2.9b can be written in the 0-d-q axes using Eqs. 2.8. This is shown in Eq. 2.10a.

$$p[i_e] = \Sigma\ p[i_k] = \Sigma\ [D_k]\ [V_B] + \Sigma\ [E_k]\ [i_k] \tag{2.10a}$$

where

$$[V_B] = [v_{qB} \quad v_{qr} \quad v_{dB} \quad v_{dr}]^T \tag{2.10b}$$

and for singly fed induction machines,

$$v_{qr} = v_{dr} = 0. \tag{2.10c}$$

Even though there is no physical connection between the the rotor circuits of all N machines, it is mathematically possible to carry out the summation of the rotor currents in Eq. 2.10a. For singly-fed machines, the impressed voltages on the rotor circuits are zero. The rotor circuits of all N machines may therefore be considered to be fed from a common ground bus. Thus, it is possible to say that the rotor current in the equivalent machine is the sum of the rotor currents of all the N machines.

Since all the machines are induction machines, the equivalent machine which would represent them would also be an induction machine. Therefore, the equations of the equivalent induction machine would have the same form as

Eqs. 2.8. The following equation can then be written for the electrical circuits of the equivalent machine.

$$p[i_e] = [D_e]\ [V_B] + [E_e]\ [i_e] \qquad (2.11a)$$

where $[D_e]$ and $[E_e]$ are the corresponding [D] and [E] matrices of the equivalent machine.

Equation 2.11a must be equivalent to Eq. 2.10a if the equivalent machine is to represent the total effect of all the individual induction machines. For equivalence between Eq. 2.10a and Eq. 2.11a,

$$[D_e] = \Sigma\ [D_k] \qquad (2.11b)$$

$$[E_e]\ [i_e] = \Sigma\ [E_k]\ [i_k] \qquad (2.11c)$$

In order to obtain the parameters for the equivalent machine, it is necessary to find expressions for the elements of the $[E_e]$ matrix so that Eq. 2.11c is satisfied. A solution is found by carrying out a term-by-term comparison of the [E] [i] products in Eq. 2.11c. Expanding Eq. 2.11c gives Eq. 2.12a, which may also be written as shown in Eq. 2.12b.

$$\begin{bmatrix} (E_{11})_e & (E_{12})_e & \cdots \\ (E_{21})_e & (E_{22})_e & \cdots \\ \cdot & \cdot & \\ \cdot & \cdot & \end{bmatrix} \begin{bmatrix} (i_{qs})_e \\ (i_{qr})_e \\ (i_{ds})_e \\ (i_{dr})_e \end{bmatrix} = \Sigma \begin{bmatrix} (E_{11})_k & (E_{12})_k & \cdots \\ (E_{11})_k & (E_{12})_k & \cdots \\ \cdot & \cdot & \\ \cdot & \cdot & \end{bmatrix} \begin{bmatrix} (i_{qs})_k \\ (i_{qr})_k \\ (i_{ds})_k \\ (i_{dr})_k \end{bmatrix} \qquad (2.12a)$$

In the matrices of Eq. 2.12a, the terms $(E_{11})_k$, $(E_{12})_k$, etc. refer to the various elements in the [E] matrix of the k-th machine.

A similar notation (with subscript e) is used for the [E] matrix of the equivalent machine. The [E] matrices are 4 X 4 in order and the expressions for the elements are given in Eq. 2.8e. Equation 2.12a may be expanded and rewritten as shown in Eq. 2.12b.

$$\begin{bmatrix} (E_{11})_e(i_{qs})_e & (E_{12})_e(i_{qr})_e\cdots \\ (E_{21})_e(i_{qs})_e & (E_{22})_e(i_{qr})_e\cdots \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \Sigma \begin{bmatrix} (E_{11})_k(i_{qs})_k & (E_{12})_k(i_{qr})_k\cdots \\ (E_{21})_k(i_{qs})_k & (E_{22})_k(i_{qr})_k\cdots \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{2.12b}$$

Each matrix in Eq. 2.12b is multiplied by the same vector, hence, term-by-term comparison is possible in this equation. This comparison gives the results of Eq. 2.12c.

$$\begin{aligned} (E_{11})_e\ (i_{qs})_e &= \Sigma\ (E_{11})_k\ (i_{qs})_k \\ (E_{12})_e\ (i_{qr})_e &= \Sigma\ (E_{12})_k\ (i_{qr})_k \\ &\vdots \\ (E_{44})_e\ (i_{dr})_e &= \Sigma\ (E_{44})_k\ (i_{dr})_k \end{aligned} \tag{2.12c}$$

Expressions can then be obtained for the elements of the $[E_e]$ matrix from Eq. 2.12c as shown in Eq. 2.13.

Equations 2.11b and 2.13 give the elements of the [D] and [E] matrices of the equivalent induction machine. The

$$(E_{11})_e = \Sigma\ (E_{11})_k\ (i_{qs})_k/(i_{qs})_e$$
$$(E_{12})_e = \Sigma\ (E_{12})_k\ (i_{qr})_k/(i_{qr})_e$$
$$\vdots$$
$$(E_{44})_e = \Sigma\ (E_{44})_k\ (i_{dr})_k/(i_{dr})_e \quad (2.13)$$

values of the currents and voltages are given in Eqs. 2.9 and 2.10. Equations 2.13 can be approximated in terms of the machine kVA loadings if it is assumed that all the machines operate at the same power factor. This approximation is developed in Appendix D and is given as

$$(i_{qs})_k/(i_{qs})_e = (i_{ds})_k/(i_{ds})_e = (kVA)_k/(kVA)_e \quad (2.14)$$

where

$(kVA)_k$ = the kilovoltampere drawn by the k-th machine

$(kVA)_e$ = the total kVA drawn by all machines

A similar assumption for the rotor power factors yield the same kVA ratios given in Eq. 2.14 for the ratios of the rotor currents in Eq. 2.13. The elements of the $[E_e]$ matrix may, therefore, be computed as shown in Eq. 2.15.

$$(E_{11})_e = \Sigma\ (E_{11})_k\ (kVA)_k/(kVA)_e$$
$$(E_{12})_e = \Sigma\ (E_{12})_k\ (kVA)_k/(kVA)_e$$
$$\vdots$$
$$(E_{44})_e = \Sigma\ (E_{44})_k\ (kVA)_k/(kVA)_e \quad (2.15)$$

The elements of the $[E_e]$ matrix are, therefore, approximately equal to the weighted sums of the corresponding elements of the [E] matrices of the individual machines, the weighting factors being the kilovoltamperes drawn by the machines. If the machines do not operate at the same power factor but between power factor ranges of 65% and 91%, it is shown in Appendix D that Eq. 2.15 gives results that are within 2.5% of those obtained from Eq. 2.13. It is not necessary to use Eq. 2.15, which is only approximate, since the more accurate Eq. 2.13 could easily be used. However, since it is expressed in kVAs, Eq. 2.15 gives a more readily recognizable form for the weighting of the elements of the [E] matrix.

D. Parameters of the equivalent induction machine

From the matrix equations of Eq. 2.8e and Eq. 2.8i, the parameters of the induction machine may be obtained by manipulating the elements of the matrices as shown in Eqs. 2.16.

$$R_s' = -E_{21}/D_{21} \tag{2.16a}$$

$$R_r = -E_{12}/D_{21} \tag{2.16b}$$

$$L_{rr} = -D_{11}M/D_{21} \tag{2.16c}$$

$$L_{ss}' = -E_{22}M/E_{12} \tag{2.16d}$$

M = the mutual inductance between stator and the rotor circuits

The equations for the equivalent induction machine have the same form as the equations for a single induction machine. Hence, Eqs. 2.16 also apply to the matrices of the equivalent machine. The parameters of the equivalent induction machine may be obtained by using the corresponding elements of the $[D_e]$ and $[E_e]$ matrices in Eqs. 2.16. These equations require the value of the mutual inductance of the equivalent induction machine. This could be determined as outlined below.

From Eq. 2.9a, it is shown that the current drawn by the equivalent induction machine is the sum of the currents drawn by the individual machines. Also, the torque developed by the equivalent machine is shown in Section F to be the sum of the torques developed by the individual machines. The electrical torque developed by any induction machine is shown in Appendix A to be

$$T_e = M(i_{ds}i_{qr} - i_{dr}i_{qs}) \tag{2.16e}$$

The mutual inductance (M_e) between the stator and the rotor circuits of the equivalent induction machine must have a value such that the torque developed by the equivalent machine satisfies the total torque criterion mentioned above. Therefore,

$$(T_e)_{equiv.} = \Sigma\ (T_e)_k = M_e(i_{dse}i_{qre} - i_{dre}i_{qse}) \tag{2.16f}$$

where

$(T_e)_{equiv.}$ = electrical torque of the equivalent machine

$(T_e)_k$ = electrical torque of the k-th machine

From Eq. 2.16f,

$$M_e = (T_e)_{equiv.}/(i_{dse}i_{qre} - i_{dre}i_{qse}) \qquad (2.16g)$$

The speed of the equivalent machine, $(\omega_r)_e$, may be obtained from the equation relating the torque and the mechanical power output. In the steady state, the expression for the speed is shown in Eq. 2.16h.

$$(\omega_r)_e = \Sigma\ (T_e)_k(\omega_r)_k/(T_e)_{equiv.} \qquad (2.16h)$$

E. The Inertia of the Equivalent Induction Machine and Its Mechanical Load

Associated with the rotation of each machine is some amount of kinetic energy. For the k-th machine, this energy is given by

$$W_k = H_k\ S_{bk} \qquad (2.17a)$$

where

W_k = kinetic energy of k-th machine

H_k = inertia of the k-th machine

S_{bk} = voltampere rating of the k-th machine

The total kinetic energy ,W_e, of all the N machines is given by

$$W_e = \Sigma W_k = \Sigma H_k S_{bk} \tag{2.17b}$$

The inertia of the equivalent machine is chosen such that total kinetic energy is conserved. Therefore,

$$W_e = H_e S_{be} = \Sigma H_k S_{bk} \tag{2.17c}$$

and, hence,

$$H_e = \Sigma H_k S_k / S_{be} \tag{2.17d}$$

where

H_e = inertia of equivalent machine

S_{be} = voltampere base of the equivalent machine

F. Mechanical Torque of the Equivalent Machine

The per unit output power of the k-th machine is

$$P_k = (T_m)_k (\omega_r)_k \tag{2.18}$$

where $(T_m)_k$ is the mechanical torque on the k-th machine and $(\omega_r)_k$ its speed. The total output power of all machines is

$$P_t = \Sigma P_k = \Sigma (T_m)_k (\omega_r)_k \tag{2.19}$$

For the equivalent machine, the power output is

$$P_e = (T_m)_e (\omega_r)_e \tag{2.20a}$$

where $(T_m)_e$ is the mechanical torque on the equivalent machine and $(\omega_r)_e$ its speed. P_e must equal P_t for conservation of mechanical power. Therefore,

$$(T_m)_e(\omega_r)_e = \Sigma\ (T_m)_k(\omega_r)_k \qquad (2.20b)$$

The per unit speeds of the machines are usually close to unity. To obtain an expression for $(T_m)_e$, consider the limiting case where $(\omega_r)_k = 1.0$ for all k and, hence, the speed $(\omega_r)_e$ of the equivalent machine is also 1.0. Substituting these values in Eq. 2.20b gives

$$(T_m)_e = \Sigma\ (T_m)_k \qquad (2.21)$$

The mechanical torque equals the electrical torque in the steady state. Therefore, Eq. 2.21 also holds for the electrical torque of the machines.

G. Parameters of the Equivalent Mechanical Load

The torque developed by the mechanical load usually varies with its speed. If the speed of the mechanical load is ω_r, the variation of torque with speed may be described by Eq. 2.22.

$$T_m = a + b\omega_r + c\omega_r^{\,2} + d\omega_r^{\,\beta} \qquad (2.22)$$

Some of the coefficients (a, b, c, d) but not all, could equal zero. If all the torque coefficients are converted to the same kVA base (the conversion is shown in Appendix C), then using the conservation of torque criterion developed in Eq. 2.21, the total torque on all the machines is given by

$$\Sigma\ (T_m)_k = \Sigma\ \{a_k + b_k(\omega_r)_k + c_k(\omega_r)_k^{\ 2} + d_k(\omega_r)_k^{\ \beta k}\} \qquad (2.23)$$

This total torque should equal the mechanical torque on the equivalent machine as given in Eq. 2.24.

$$(T_m)_e = a_e + b_e(\omega_r)_e + c_e(\omega_r)_e^{\ 2} + d_e(\omega_r)_e^{\ \beta e} \qquad (2.24)$$

Term-by-term comparison between Eq.2.23 and Eq. 2.24 yields the torque coefficients of the equivalent mechanical load which are given in Eqs. 2.25.

$$a_e = \Sigma\ a_k \qquad (2.25a)$$

$$b_e = \Sigma\ b_k(\omega_r)_k/(\omega_r)_e \qquad (2.25b)$$

$$c_e = \Sigma\ c_k(\omega_r)_k^{\ 2}/(\omega_r)_e^{\ 2} \qquad (2.25c)$$

$$d_e(\omega_r)_e^{\ \beta e} = \Sigma\ d_k(\omega_r)_k^{\ \beta k} \qquad (2.25d)$$

Equation 2.25d holds for the per unit operating speeds of the loads. These speeds are near 1.0 per unit. Consider the limiting case when $(\omega_r)_k = 1.0$ for all machines (and loads), then the speed $(\omega_r)_e$ of the equivalent machine (and load) also equals 1.0. For this limit,

$$d_e = \Sigma\ d_k \qquad (2.25e)$$

The value of βe could then be determined from Eq. 2.25d and the actual operating speeds to be

$$\beta e = \log[\Sigma\ (d_k/d_e)(\omega_r)_k^{\beta k}]/\log[(\omega_r)_e]. \qquad (2.25f)$$

H. Summary

Differential equations are written in the 0-d-q axes for a number of induction machines connected to a common bus. These equations are used to develop the equations of an equivalent induction machine that would represent all the individual machines and their feeders. The parameters of the equivalent machine are determined from its equations.

The equivalent machine draws the same amount of current from the source as drawn by all the individual machines it replaces. It also develops torque equal to the sum of the torque developed by the individual machines. Furthermore, the equivalent machine drives a mechanical load which is an equivalent of the mechanical loads driven by the individual machines. It delivers the same amount of power to the equivalent mechanical load as the sum of the power delivered by all the machines it replaces.

III. MODEL RESPONSE AND VALIDATION

A. Introduction

A method for finding the equivalent of two or more induction machines connected to the same bus has been developed in the previous chapter. This method is applied to the three induction machines shown in Fig. 3.1. The three machines are rated at 2300 V and 800 hp, 800 hp and 1000 hp. The parameters of the machines are listed in Table 3.1. The values are in per unit on the machine kVA and voltage bases. All feeders are assumed to have negligible impedances.

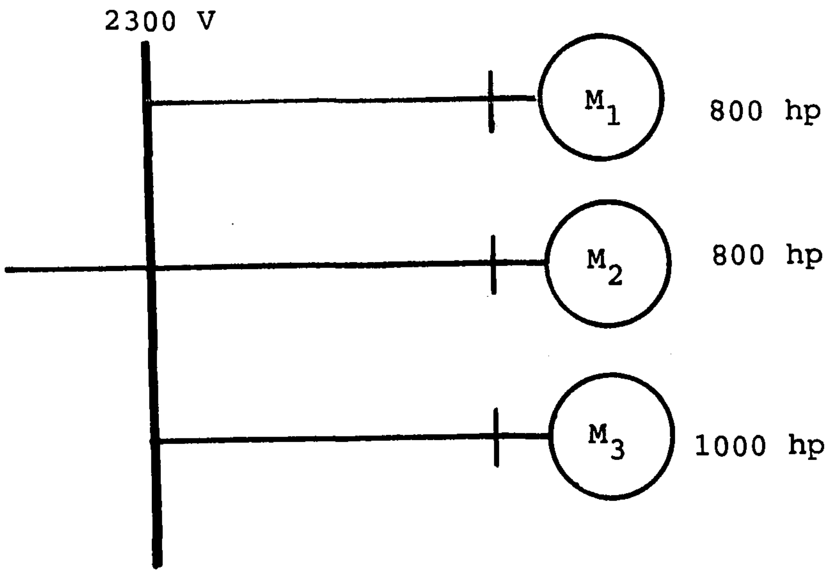

Fig. 3.1. Three induction machines connected to a common bus

An induction machine that is equivalent to all three machines is found using the method developed in Chapter II. Its parameters are listed in Table 3.1. The rated voltage of the equivalent machine is chosen to be the voltage at the common bus - 2300 V. The equivalent machine is rated at 2250 kVA, which is the sum of the kVA ratings of the three machines. However, it may sometimes be desirable to express the parameters of the equivalent machine on different voltage and kVA bases (e.g., the base kVA used in a study). In such cases, the conversion formulas shown in Appendix B may be used. The parameters that describe the torque developed by the mechanical loads connected to the machines are also listed in Table 3.1.

When feeder impedances are significant, the parameters computed for the equivalent induction machine will be different from those computed by assuming negligible feeder impedances. As an example, the parameters of these same three machines and their equivalent when they are supplied through feeders that have high impedances are listed in Table 3.2. Each feeder is assumed to have an impedance of 0.01 + j0.1 per unit based on the machine voltage and kVA bases.

In order to carry out any studies on the three machines, it is necessary to simulate the differential equations that describe their responses. These equations

Table 3.1. The parameters of three induction machines and their equivalent machine

Parameter	Values (per unit on machine base)			
	Machine 1	Machine 2	Machine 3	Equivalent Machine
Rating horsepower (kVA)	800 (700)	800 (700)	1000 (850)	2600 (2250)
R_s	0.012	0.015	0.016	0.0145
X_s	0.120	0.081	0.085	0.0954
R_r	0.010	0.011	0.010	0.0103
Xr	0.072	0.081	0.085	0.0775
X_m	3.200	4.070	7.634	4.4960
H	0.490	0.633	0.711	0.6180
Load coefficients				
a	0.0	0.0	0.1	0.0378
b	0.1	0.1	0.3	0.1755
c	0.2	0.3	0.0	0.1556
d	0.6	0.5	0.6	0.5689
β	2.3	1.9	1.5	1.8581
Feeder impedance	0	0	0	0

Table 3.2. The parameters of three induction machines, their feeders and their equivalent machine

Parameter	Values (per unit on machine base)			
	Machine 1	Machine 2	Machine 3	Equivalent Machine
Rating horsepower (kVA)	800 (700)	800 (700)	1000 (850)	2260 (2250)
R_s	0.012	0.015	0.016	0.0245
X_s	0.120	0.081	0.085	0.1958
R_r	0.010	0.011	0.010	0.0103
X_r	0.072	0.081	0.085	0.0779
X_m	3.200	4.070	7.634	4.5056
H	0.490	0.633	0.711	0.6180
Load coefficients				
a	0.0	0.0	0.1	0.0378
b	0.1	0.1	0.3	0.1755
c	0.2	0.3	0.0	0.1556
d	0.6	0.5	0.6	0.5689
β	2.3	1.9	1.5	1.8632
Feeder impedance	0.01+j0.1	0.01+j0.1	0.01+j0.1	0.0+j0.0

are shown in Appendix A. The equations are of fifth order. Therefore, fifteen differential equations will be required to fully simulate all three machines. However, if the equivalent machine is used, only five differential equations are required.

B. Comparison of the Responses of Three Induction Machines and Their Equivalent Machine to Voltage Dips

Most major disturbances that occur in electric power systems are voltage disturbances. These are mainly due to faults in the power system. The voltage is known to drop by as much as 70% from its nominal value at buses near which faults occur. To test the accuracy of the equivalent machine, a simulation of the response of the equivalent machine to a voltage disturbance is carried out. This is compared with the 'actual' response of the three machines.

The 'actual' response of the three machines is obtained by carrying out simulations on the various machines individually and summing up the responses. For example, to obtain the 'actual' value of the total current at any instant of time, the instantaneous value of the current drawn by each machine is calculated for that instant of time and the total found as an addition of the three currents. A fifth order model is used for each machine. For the three machines studied, this requires the simulation of fifteen differential equations.

A 70% voltage dip that lasts for 0.1 seconds is introduced as a disturbance at the common bus of the three machines shown in Fig. 3.1. The sum of the responses of the three machines is simulated and compared with the simulation results from the equivalent machine when it is subjected to the same voltage disturbance. The variations of three quantities, namely, the active power, the reactive power and the total current in the three machines are compared with the variations of these quantities in the equivalent machine.

Figures 3.2 - 3.5 show the comparison of the responses. In Fig. 3.2 for example, the power drawn by each of the three machines during the transients is individually computed. The total power consumed is then found as the sum of the power consumed by the three machines. This sum is assumed to be the true value of the power variations. The variations in the power drawn by the equivalent machine are also plotted alongside for the comparison. The two graphs are practically coincident. Similar additions and comparisons are made for the reactive power and currents. These comparisons are depicted in Figs. 3.4 and 3.5.

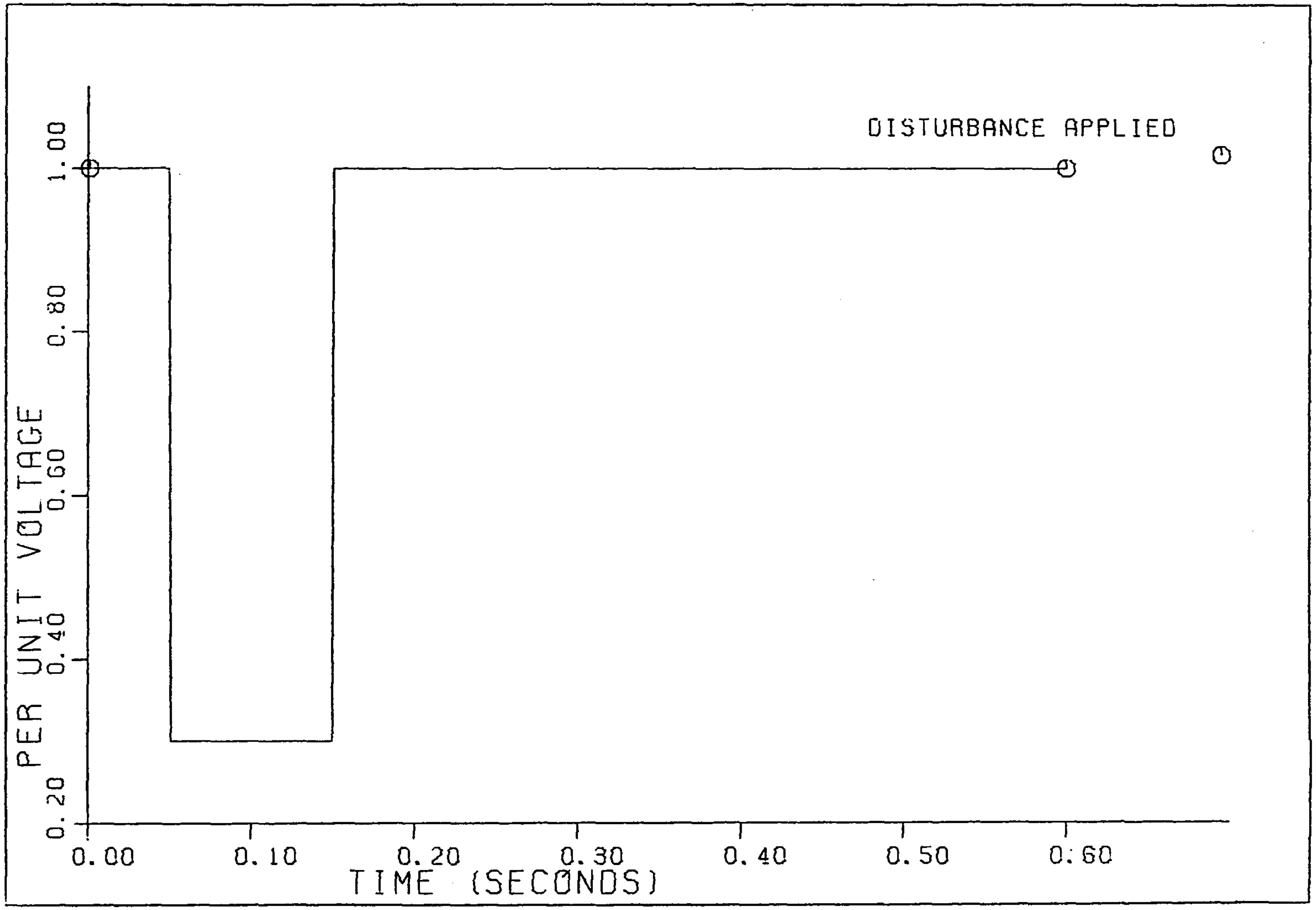

Fig. 3.2. Voltage disturbance applied to the three machines

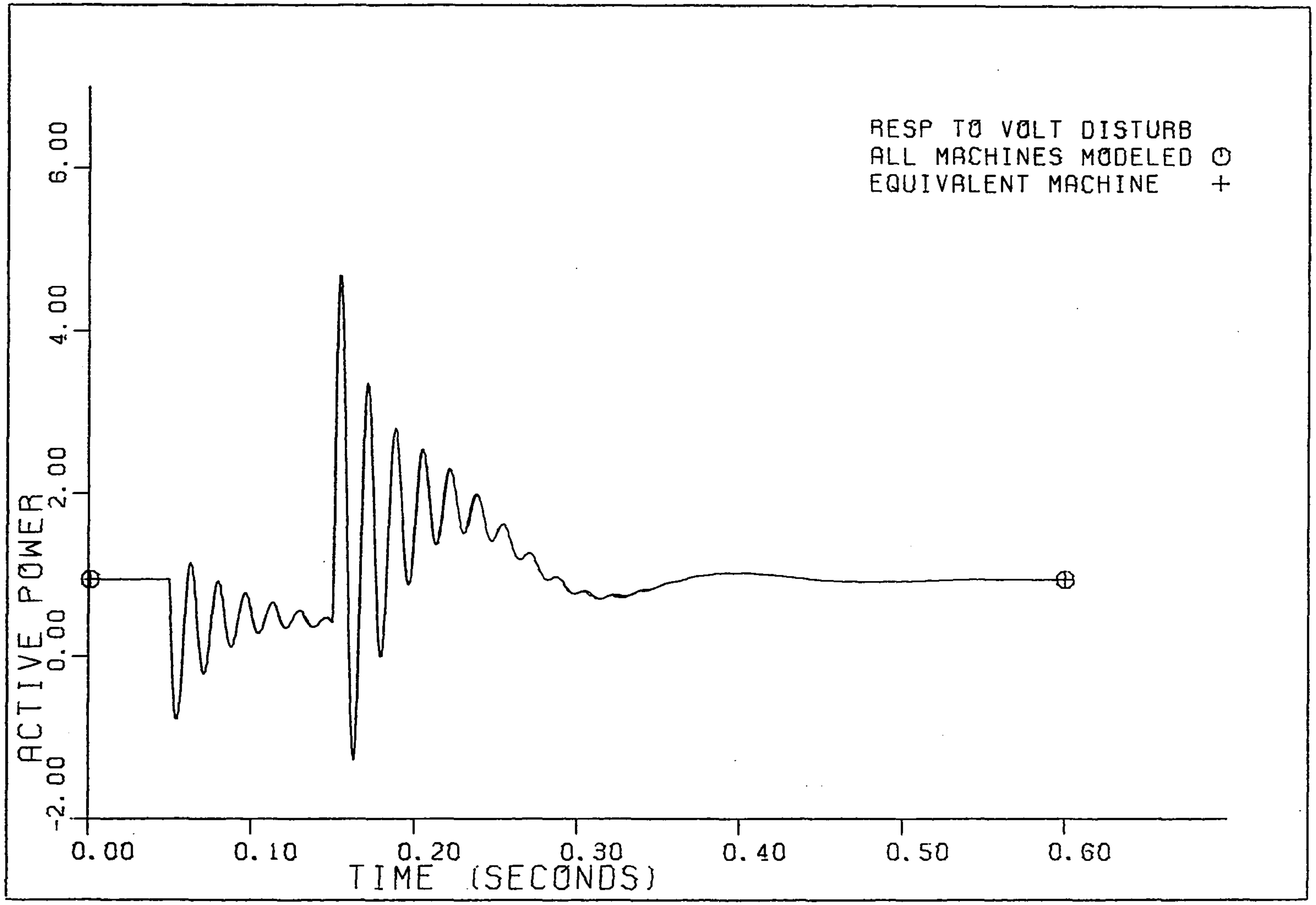

Fig. 3.3. Variations of the total active power drawn by the three induction machines and by the equivalent machine

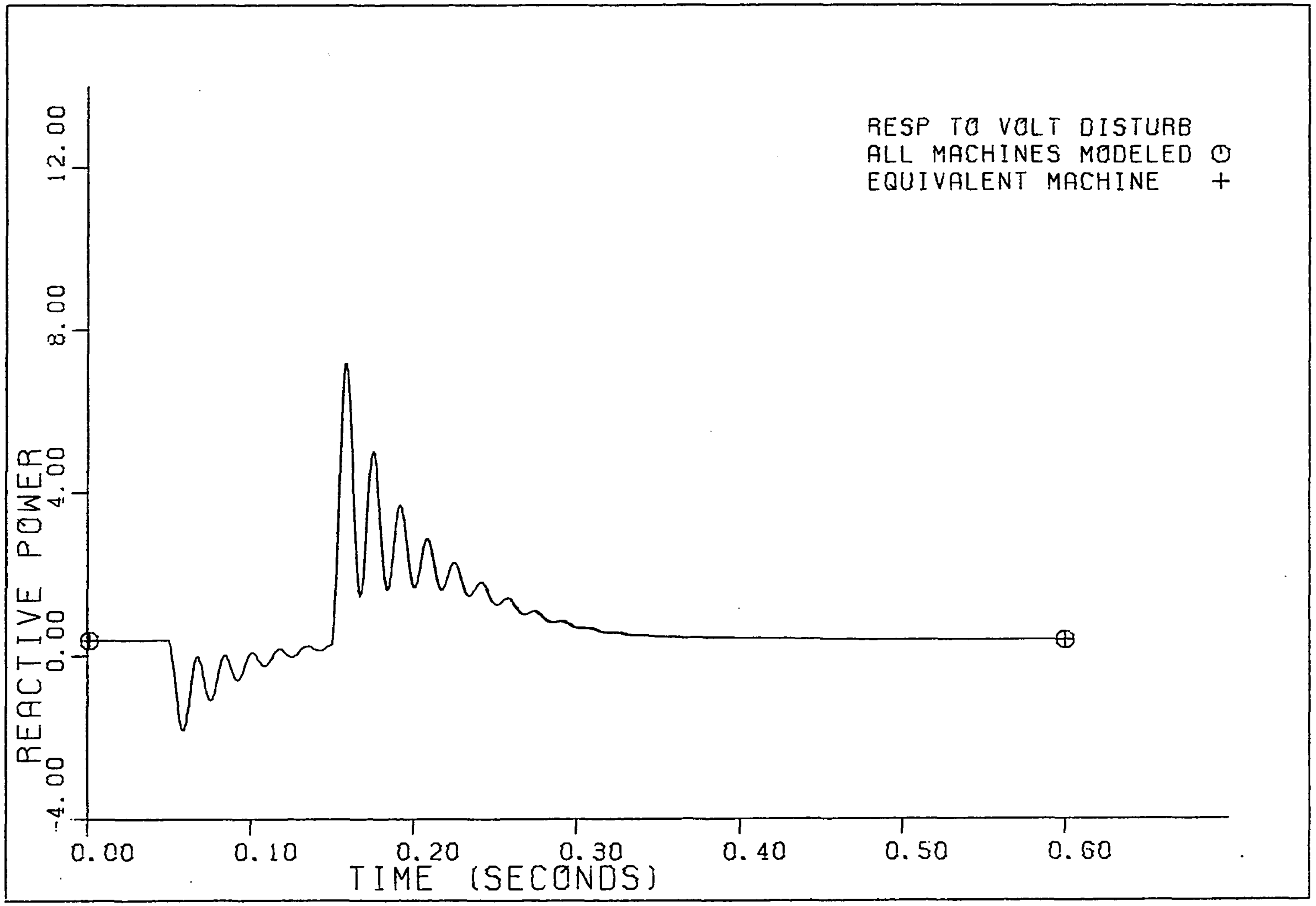

Fig. 3.4. Variations of the total reactive power drawn by the three induction machines and by the equivalent machine

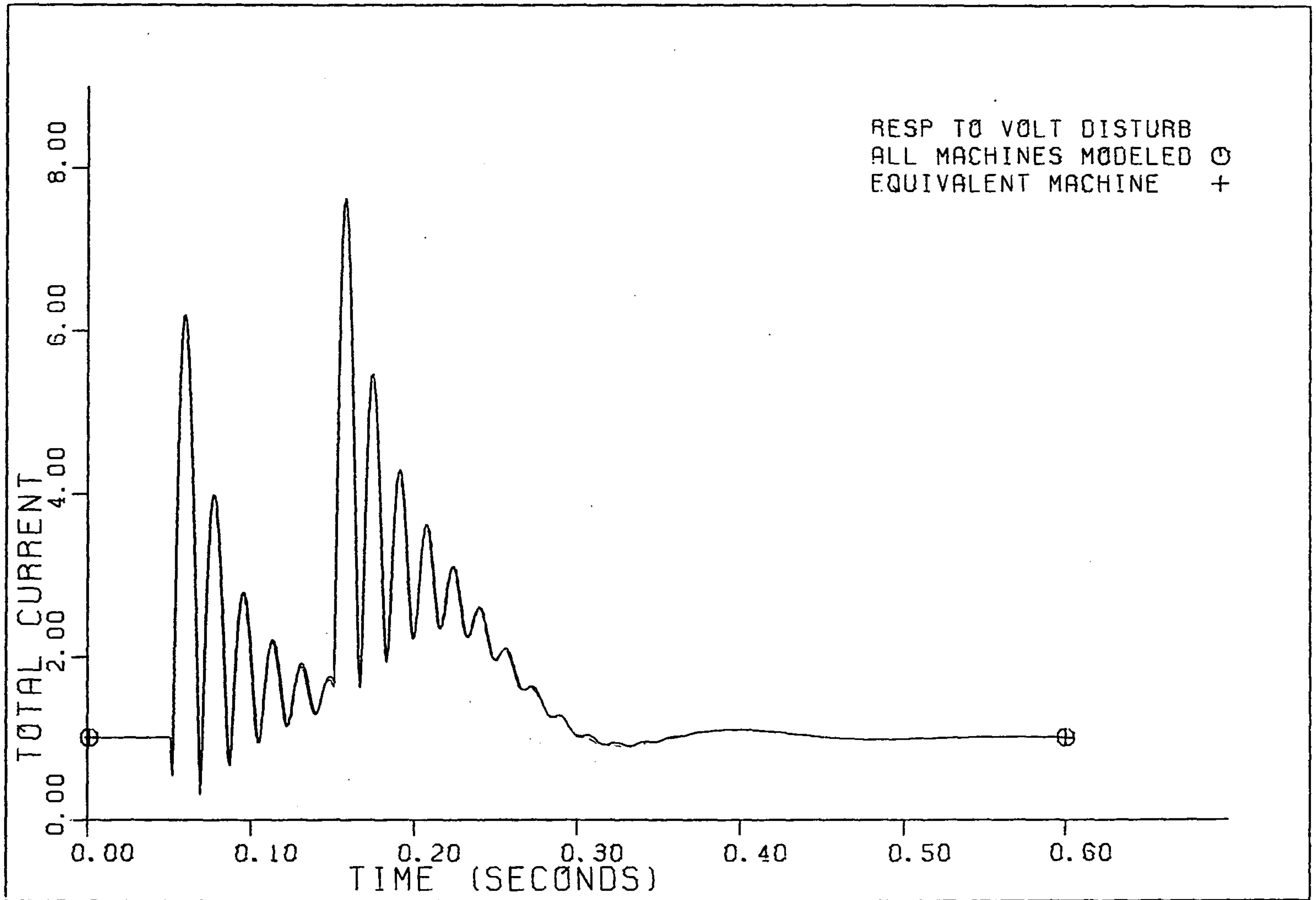

Fig. 3.5. Variations of the total current drawn by the three induction machines and by the equivalent machine

C. Comparison of the Responses of Three Induction Machines and Their Equivalent Machine to Frequency Deviations

Most electric power systems are interconnected with other power systems. The increase in size of the resulting system and the governors on the generators prevent the whole system from suddenly speeding up or slowing down beyond certain limits during disturbances. Therefore, the frequency of most power systems changes very little from its nominal value. In most power systems, the frequency changes by less than two percent (i.e. 1.2 Hz in a 60-Hz system) during faults or other major disturbances.

The variations in the current and power drawn by the three machines caused by abrupt changes in frequency are compared in Figs. 3.6 - 3.9. The frequency is changed in two steps from its nominal value of 60 Hz to 59 Hz and then back to 60 Hz. It is then increased to 61 Hz in two steps and restored to 60 Hz again after some further variations. Figure 3.6 shows the frequency deviations applied to the machines. The comparison is again between the equivalent machine and the three machines described in Table 3.1.

D. Analysis of Results

From the graphs of the simulation results shown in Figs. 3.3 - 3.9, it could be seen that the equivalent machine accurately predicts the net effect of all the

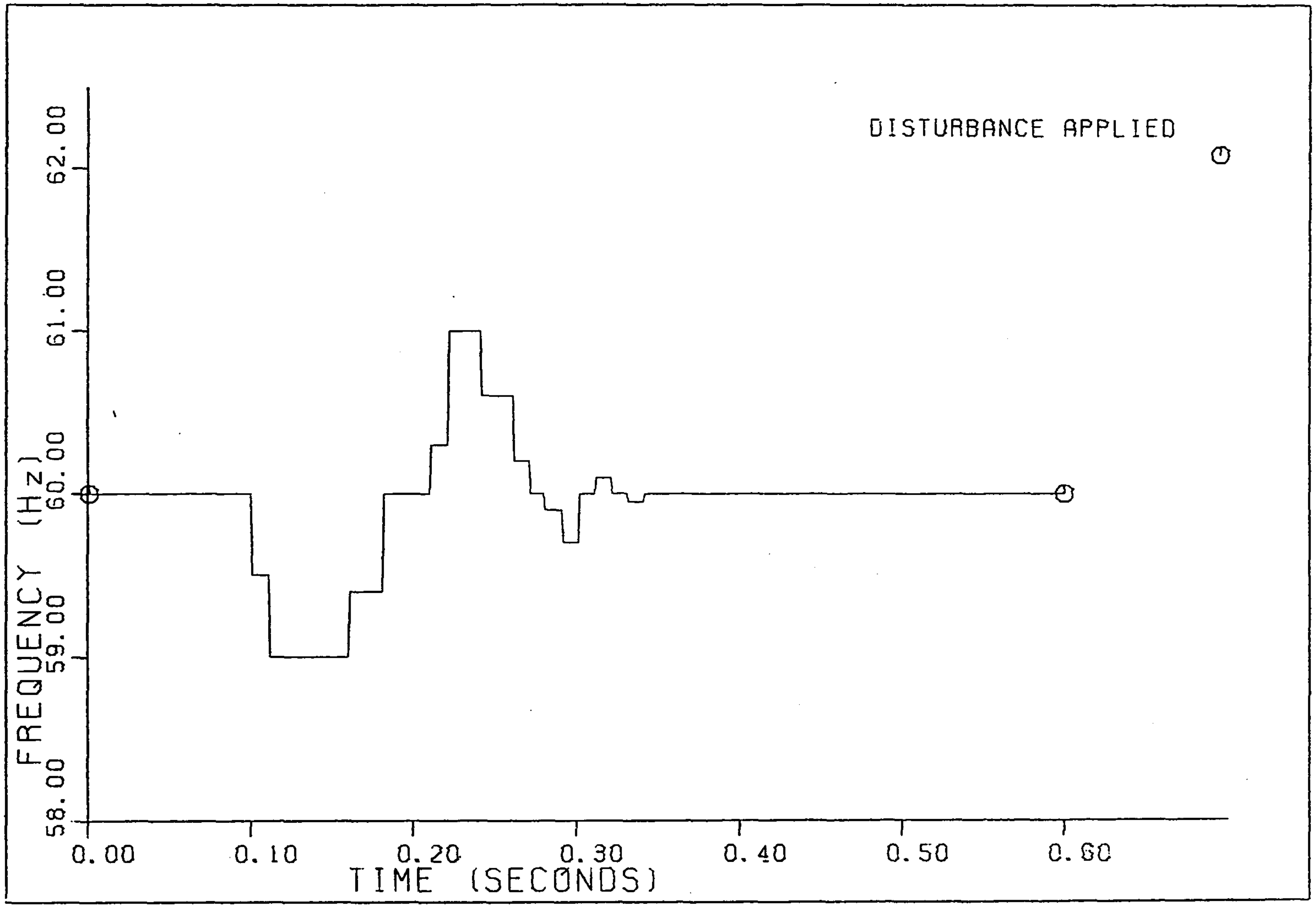

Fig. 3.6. The frequency deviations applied to the three induction machines and to the equivalent machine

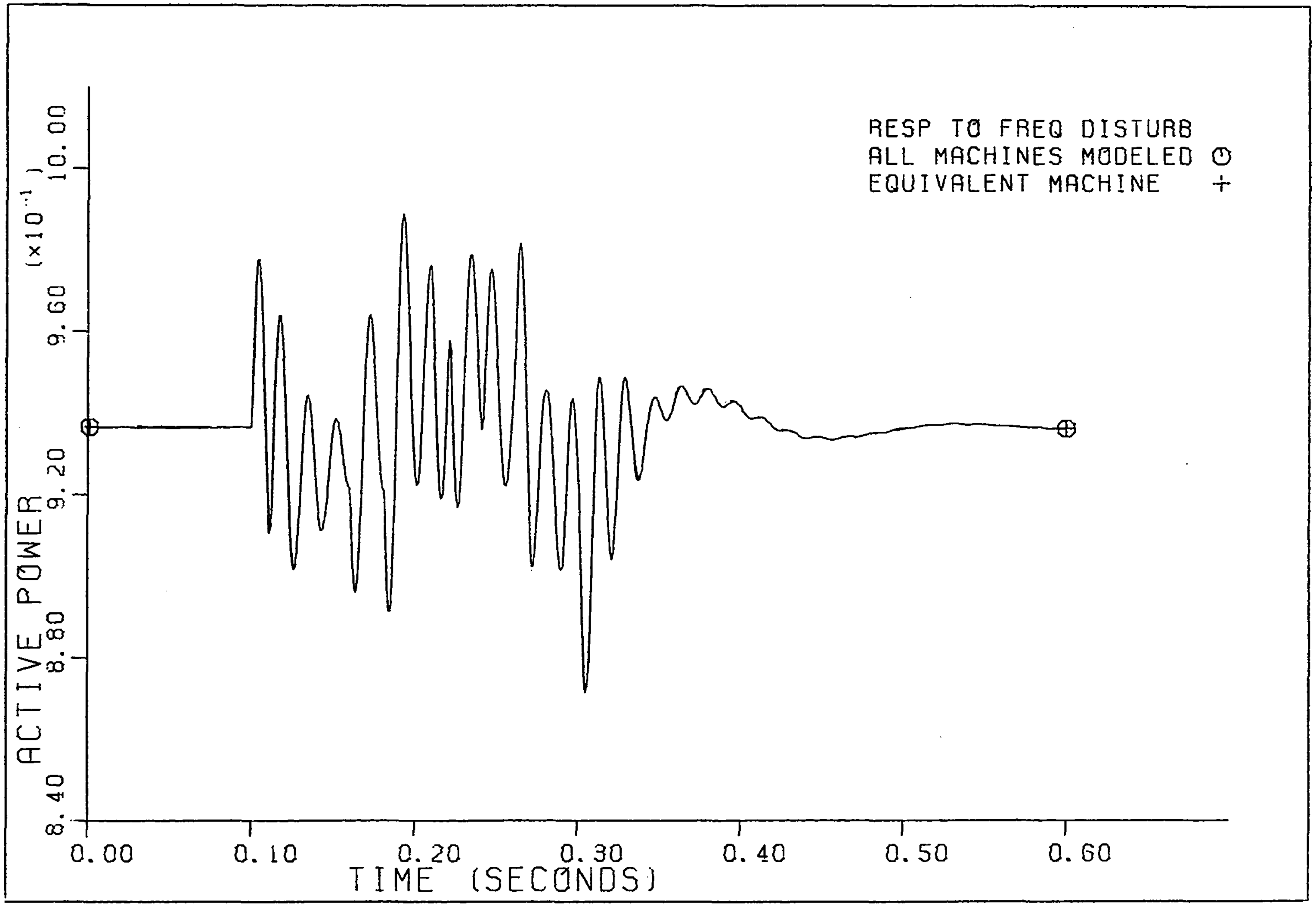

Fig. 3.7. Variations of active power drawn by three induction machines and the equivalent machine due to frequency changes

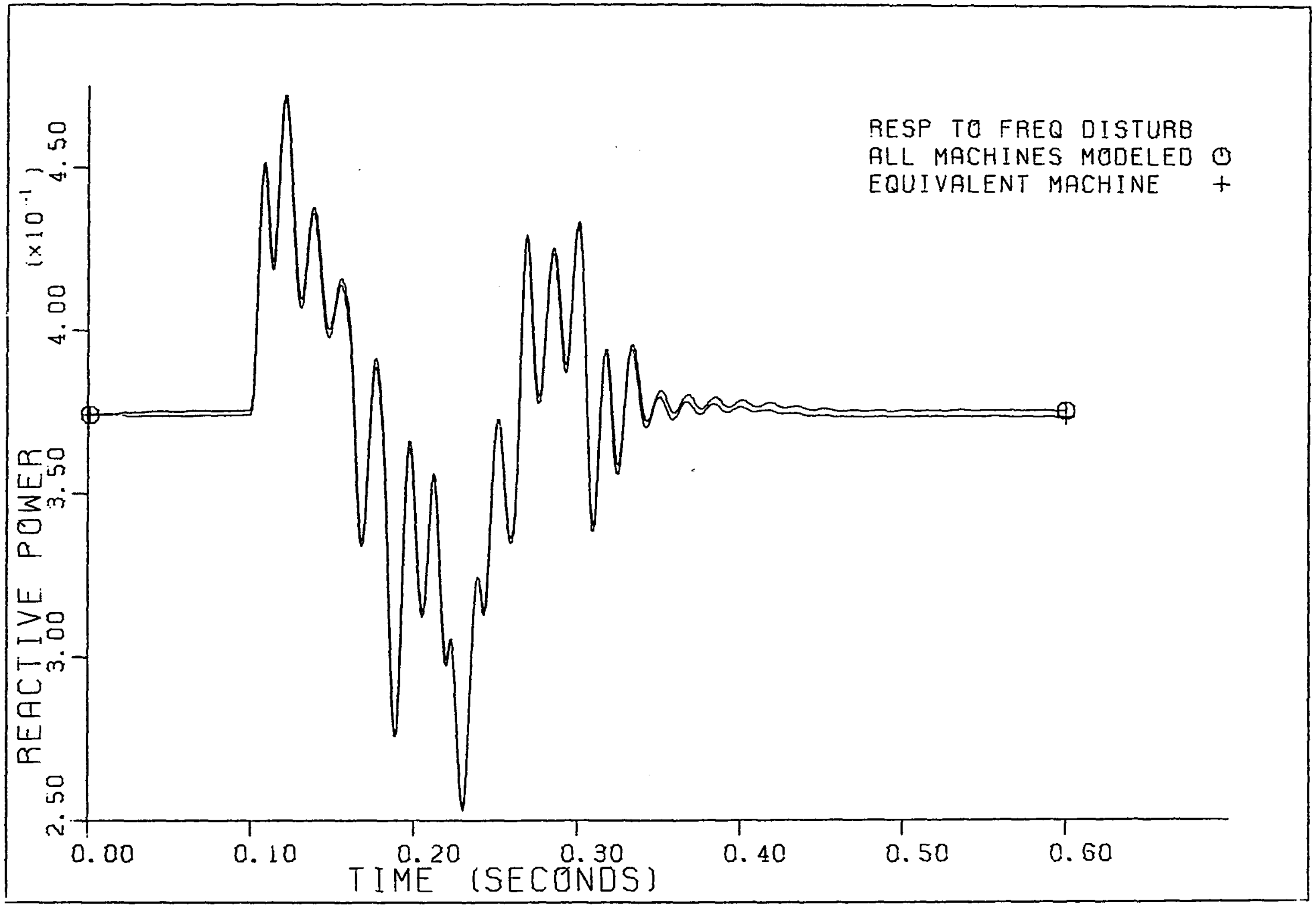

Fig. 3.8. Variations of reactive power drawn by three induction machines and the equivalent machine due to frequency changes

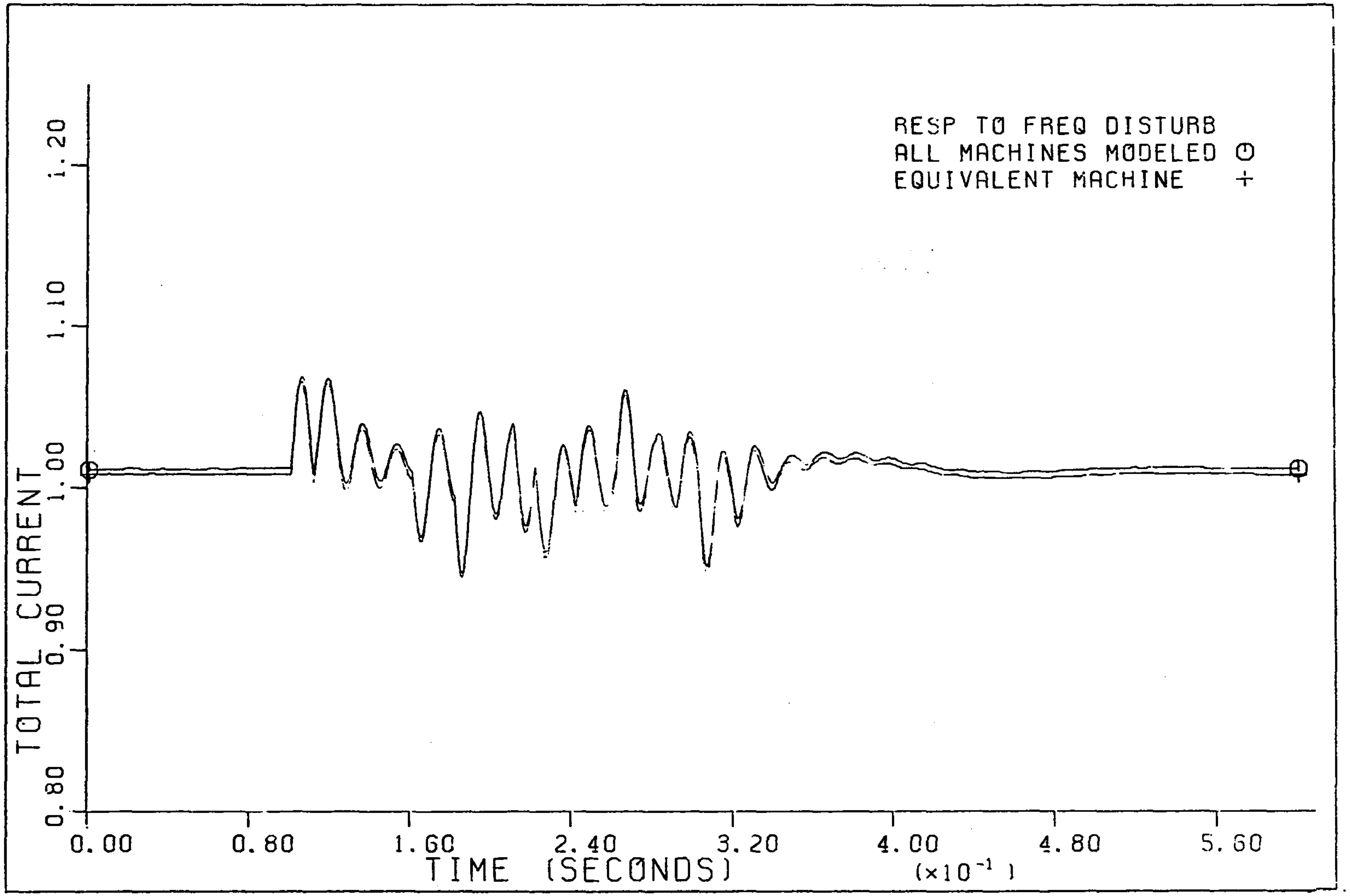

Fig. 3.9. Variations of total current drawn by three induction machines and the equivalent machine due to frequency changes

machines it replaces. For voltage and frequency disturbances, the equivalent machine has been found to closely follow all the variations in the net values of the active power, the reactive power and the current drawn by the machines.

Following the disturbances, the induction machines show transient responses that clearly indicate the presence of more than one transient phenomenon. There are the stator electrical transients which occur at or near the supply frequency. These transients are superimposed on another mode of transient oscillations called the dynamics. These dynamic oscillations are the average exchanges of energy between the machines and the supply bus. The oscillations occur at frequencies much lower than the frequency of the supply. These oscillations show as variations in the average values of the current, active and reactive power drawn by the machines.

The frequency of the dynamics in each machine listed in Table 3.1 is around 6.0 Hz. This frequency is not the same for all machines but depends mainly on the machine parameters and the operating conditions. The total inertia of the machine and its mechanical load appears to have the greatest effect on the frequency of the dynamics. Figure 3.10 illustrates the relationship between the dynamics and the stator electrical transients in the active power drawn by an induction machine.

The voltage disturbance used in the simulations produces more variations on the power oscillations than that produced by the frequency disturbances. The magnitudes of the disturbances are chosen to be close to values that might occur in practice. Voltages might change drastically during faults in power systems, but the frequency does not change very much in interconnected systems. The voltage disturbance used is 70% of the nominal value but the frequency disturbance is less than 2% of the nominal value.

E. Summary

The results of the simulations shown in this chapter clearly demonstrate the accuracy of the equivalent machine. Simulations are done for transients caused by voltage and frequency disturbances in three induction machines and their equivalent machine. The equivalent machine has been found to accurately predict the variations of the current, active and reactive power in both the steady state and during transients.

The two main components of the transient response that are of interest are the stator electrical transients and the machine dynamics. As shown by the results of the simulation, the equivalent machine clearly predicts both.

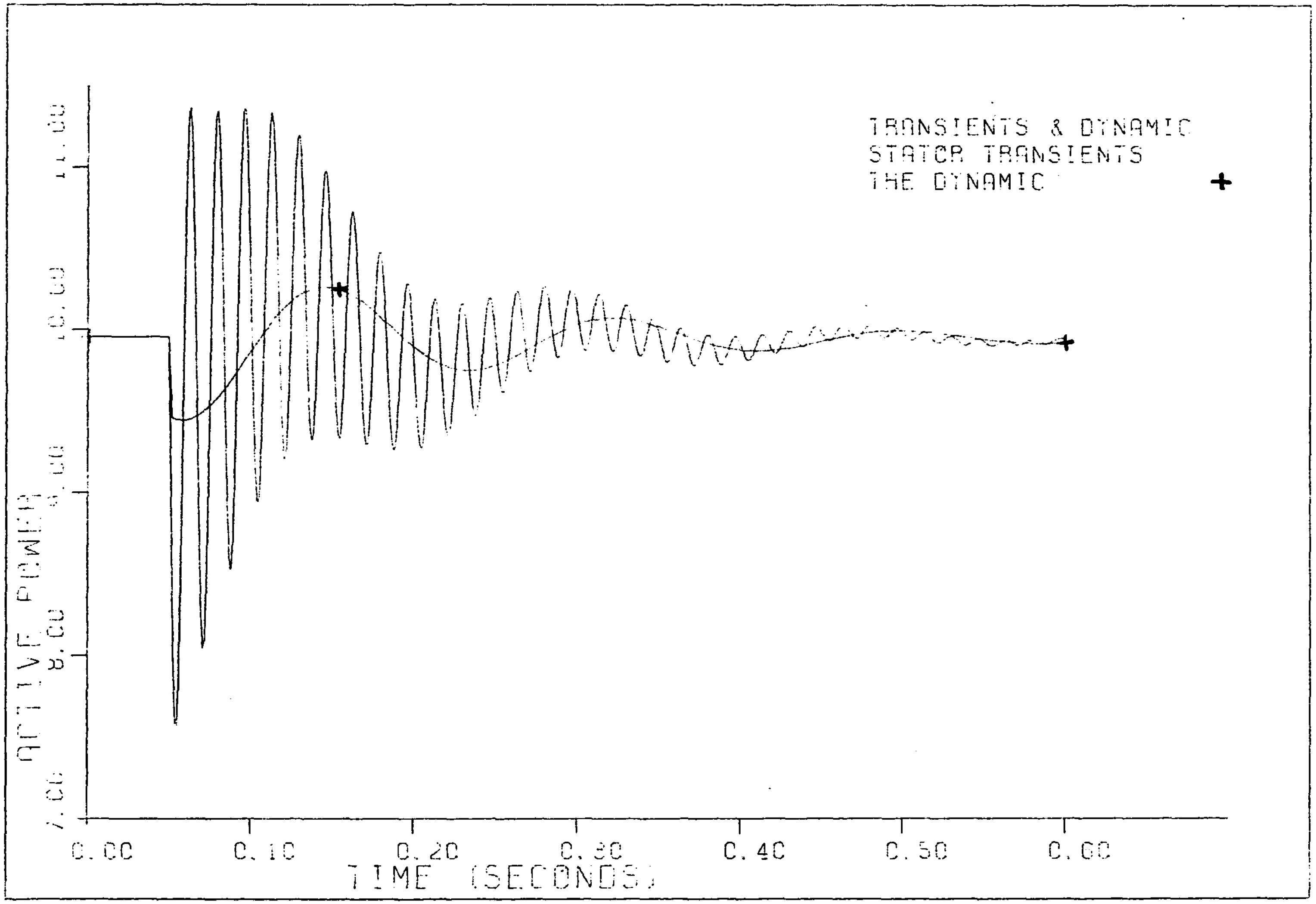

Fig. 3.10. **Relationship between the stator electrical transients and the dynamics in the active power drawn by an induction machine**

The simulations show that the dynamics, which are oscillations of energy between the machines and the power system, occur at frequencies much lower than the supply frequency.

The parameters and characteristics of induction machines found in power systems vary widely. When machines of similar characteristics are grouped together, the equivalent predicts the variations in the total power and current better than when machines of widely different characteristics are grouped together. Chapter IV discusses this in more detail and suggests two criteria for reducing any inaccuracies that might occur in grouping machines.

IV. CRITERIA FOR GROUPING OF MACHINES

A. Introduction

The response of an induction machine to power system disturbances depends on the machine parameters, the operating conditions and the inertia of the machine and its connected mechanical load. The response includes electrical transients in both the stator and the rotor, oscillations of energy between the machine and the power system, and mechanical oscillations of the rotor around the steady state speed. In general, the electrical and mechanical transients in big induction machines last longer than those in small machines.

The energy oscillations that occur between a machine and the power system are called the dynamics of the machine. The frequencies of these oscillations are much lower than the frequency of the power system. In 60-Hz power systems, most induction machines experience these dynamic oscillations at frequencies between 3 Hz and 15 Hz. Small machines exhibit the higher frequencies.

It is conceivable that the induction machines in any power system would not all be operating at the same level of loading. Some may be fully loaded while others may be partially loaded or unloaded. The response of any induction machine to system disturbances is partly governed by its loading level. This also determines the contribution of that machine to an equivalent machine when it is grouped together with other machines.

B. Development of Grouping Criteria

If the frequencies of the dynamics in any two machines are not exactly equal, there will be a phase difference in the oscillations of the average power and currents drawn by these machines during transients. This means that there may be times when one machine would be taking more power from the system than it did in the steady state while the other is rejecting power into the system. An equivalent machine that would represent these two machines would give an average of these two conditions. It has been found that this average does not always represent the instantaneous values of the total power transfer between the system and any two machines during the transient period.

The equivalent machine would have only one frequency at which it experiences power oscillations. This frequency is some weighted average of the frequencies of the machines it replaces and may be different from that of any of the original machines. These differences in frequency cause phase differences between the values calculated for the power oscillations using the equivalent machine and the actual values.

Differences in phase imply that the equivalent machine would predict the peaks and valleys of the power oscillations to occur at different instances of time than actually experienced. This may introduce inaccuracies in the response of the equivalent machine. To minimize these inaccuracies in grouping two or more machines into an equivalent, it is desirable that the frequencies of power oscillation for the machines be close.

If two power oscillations of different frequencies proceed, the phase separation between them will increase with time. Assuming that the oscillations start in phase at the onset, the phase angle will eventually reach a point where the two oscillations are exactly out of phase. At this point, there will be a greater percentage error in assuming the resultant power to be in phase with any one of the oscillations.

In order to minimize any inaccuracies in grouping machines into equivalents, only those machines whose oscillations are greatly attenuated before reaching the point of 180° phase separation are suggested to be included in each group. (As will be shown shortly, this is not a major problem because most industrial induction machines can be combined into only three different groups.) It is therefore necessary to have some knowledge of the frequencies and the durations of these oscillations before attempting to group any number of machines together.

The equations that describe the responses of induction machines to voltage, frequency and load disturbances are nonlinear. By linearizing them about the machine operating points, information about the frequencies of oscillation and the durations of these oscillations could be obtained from the eigenvalues of the linearized equations.

A fifth order model for the induction machine would give five eigenvalues [28, 38], four of which are complex. The four occur as two pairs of complex conjugates. The last one is only real. The first pair of complex eigenvalues describe the electrical transients in the stator. The magnitude of the imaginary parts of this pair of complex eigenvalues is very close to the angular frequency of the power system. A second pair of complex eigenvalues describes the oscillations of energy between the machine and the power system. These oscillations are known as the machine dynamics. The real parts of these eigenvalues represent the attenuation of the oscillations and the imaginary parts denote the frequencies. The fifth eigenvalue is only real and it represents the first order inertial response of the machine.

In general, the duration of any of these transient phenomena depends on the damping associated with it.

Eigenvalues have been calculated for the the range of induction machines commonly found in practice. (An example of these is shown in Table 4.3.) They predict different attenuations (a measure of damping) and frequencies of oscillation. These different frequencies suggest that it would be necessary to use more than one equivalent machine at load buses where many different types of induction machines are connected.

A mathematical derivation of the criterion for grouping induction machines is given shortly. In the formulation of the criterion, it is assumed that the oscillations are in phase at the onset of the transients. The basis for the derivation is that by the time the phase separation reaches 180°, the amplitude of one oscillation is reduced to at least one percent of its original value.

Since the eigenvalues of the linearized equations predict the frequency and attenuation of the power oscillations, the eigenvalues are employed in the development of the criterion. This assumes that the predicted values of the frequency and attenuation, which are correct for small disturbances, would be almost equal to the values experienced during large disturbances. The eigenvalues of the machines could be obtained a priori since it only requires the knowledge of the machine parameters, the load parameters (or the loading level) and the operating voltage.

To determine whether two machines are compatible candidates for grouping, it is necessary to examine their eigenvalues that correspond to the dynamics. Consider two induction machines that exhibit different responses to power system disturbances. Let the two pairs of eigenvalues (λ_1, λ_2) that correspond to the dynamics in the two machines be given by

$$\lambda_1 = -\alpha_1 \pm j\omega_1 \qquad (4.1a)$$

$$\lambda_2 = -\alpha_2 \pm j\omega_2 \qquad (4.1b)$$

where

α_1 = attenuation constant of the dynamics of machine 1

α_2 = attenuation constant of the dynamics of machine 2

ω_1 = angular frequency of the dynamics of machine 1

ω_2 = angular frequency of the dynamics of machine 2

The dynamic responses of the two machines to a small disturbance are given by

$$f_1(t) = A_1 \exp\,(-\alpha_1 t \pm j\omega_1 t) \qquad (4.2a)$$

$$f_2(t) = A_2 \exp(-\alpha_2 t \pm j\omega_2 t) \qquad (4.2b)$$

where

$f_1(t)$, $f_2(t)$ refer to the instantaneous values of the power or the current

A_1, A_2 refer to the initial amplitudes of the power or the current

At the time (t_1) when the two oscillations are 180° out of phase

$$(\omega_1 - \omega_2)\, t_1 = \pi \qquad (4.3a)$$

hence

$$t_1 = \pi/(\omega_1 - \omega_2) \qquad (4.3b)$$

At this time, the amplitude of the response is

$$f(t_1) = A \exp(-\alpha t_1) \qquad (4.3c)$$

where f, A and α in Eq. 4.3c may refer to either of the two machines.

If the ratio of the amplitude at this time to the initial amplitude is m, then

$$\begin{aligned} m &= A \exp(-\alpha t_1)/A \\ &= \exp(-\alpha t_1) \\ &= \exp(-\alpha\pi/(\omega_1 - \omega_2)) \qquad (4.4) \end{aligned}$$

Therefore,

$$\alpha = [\ln(1/m)](\omega_1 - \omega_2)/\pi \qquad (4.5)$$

If the ratio m=0.01, which corresponds to 99% attenuation before 180° phase separation, then from Eq. 4.5 the minimum value of the attenuation constant should be

$$\alpha = [\ln(100)](\omega_1 - \omega_2)/\pi$$
$$\approx 1.47\ (\omega_1 - \omega_2) \qquad (4.6)$$

According to Eq. 4.6, if two machines are candidates for the same group, the attenuation constant of at least one machine should be greater than 1.47 times the difference in their angular frequencies. This is the first grouping criterion.

As an example, the eigenvalues of three machines whose parameters are given in Table 3.1 are listed in Table 4.1. The first pair of eigenvalues corresponds to the electrical transients in the stator circuit. Their imaginary parts are almost equal to the angular frequency of the power supply (377 rad/sec for a 60-Hz power system). These transients are almost in phase in all machines and, therefore, do not cause appreciable phase differences. The main eigenvalues of interest are those that correspond to the dynamics. These are denoted by $\lambda_{3,4}$ in Table 4.1. Examination of these eigenvalues reveal that all three machines satisfy the condition of Eq. 4.6.

The minimum value of α is obtained by using the largest difference in the frequencies of oscillation of any two

Table 4.1. Eigenvalues of the machines in Fig. 3.1 (the parameters are given in Table 3.1)

	Machine 1	Machine 2	Machine 3	Equivalent Machine
Rating hp (kVA)	800 (707.5)	800 (707.5)	1000 (850.0)	2600 (1000.0)
$\lambda_{1,2}$	-23.9±j375.8	-35.6±j374.7	-35.9±j374.9	-33.4±j375.1
$\lambda_{3,4}$	-10.3±j 41.4	-13.2±j 39.5	-11.5±j 36.6	-11.3±j39.1
λ_5	-19.2	-24.7	-21.2	-21.2

machines in Eq. 4.6. This corresponds to machines 1 and 3 in Table 4.1 and gives

$$\alpha = 1.47\ (41.4 - 36.5) = 7.2$$

This minimum attenuation is satisfied for all the machines as can be seen from the eigenvalues listed in Table 4.1 (where the minimum attenuation constant is 10.3).

Inaccuracies may also arise if the durations of the transients experienced by the machines in any group are not of the same order. These will occur mainly during the mechanical transient period. In order to reduce these inaccuracies in grouping of machines for studies during this transient period, it is necessary to determine how close the durations of the transients should be so that an equivalent machine would give practically the same results as would be obtained when all machines in the group are individually modeled.

If the transients in some machines last much longer than in the other machines, the response of a single equivalent machine may not be accurate for the whole time interval that the transients last. It will be accurate at the start of the transients when the rotors of all machines are experiencing mechanical oscillations and at the end of the transients when most rotor oscillations have ceased.

During the time interval in between, when the power oscillations in various machines begin to cease, the equivalent machine would not give accurate results. This is because the machines come to steady state at different times, and these settling times of the individual machines are not explicitly represented in the equivalent machine. In fact, the equivalent machine has its own settling time which is a weighted mean of those of the individual machines.

From results of various simulations, it has been found that a great degree of accuracy could be obtained when machines of comparable damping ratios are grouped together. The attenuation constants are used in determining these damping ratios. It has been determined that machines having attenuation constant ratios of less than 3 to 1 give acceptable results when grouped together into an equivalent. Equation 4.7 gives this ratio for any two machines.

$$1/3 < \alpha_1/\alpha_2 < 3 \qquad (4.7)$$

This is the second criterion for grouping induction machines. Equations 4.6 and 4.7 constitute the grouping criteria. Any two machines whose eigenvalues satisfy these criteria could be included in the same group.

C. Analysis of the Grouping Criteria

Figures 4.1 - 4.4 illustrate the necessity of the grouping criteria developed in this chapter. Six induction machines, whose parameters are given in Table 4.2, are considered. The responses of all six machines to a voltage disturbance are calculated and added together. The response of a one-machine equivalent is compared with it in Figs. 4.2 - 4.4. It is clear from the figure that the one-machine equivalent is accurate both at the beginning of the transients and in the steady state.

The eigenvalues of the machines are listed in Table 4.2. The second pair of complex eigenvalues listed for each machine in Table 4.2 correspond to the dynamics of the machine. Examination of these eigenvalues reveals that the criteria of Eq. 4.6 and Eq. 4.7 are not satisfied if all the six machines are considered together.

From the grouping criteria of Eq. 4.6 and Eq. 4.7, two equivalent machines would be required to represent all six machines for any studies that require accurate knowledge of the response of the machines during the whole period of the disturbance. The first three machines in Table 4.2 satisfy these criteria and would be put in one group. The last three also combine into another equivalent machine. The sum of the responses of the two equivalent machines are compared with that of the six machines in Figs. 4.5 - 4.7. The same

Table 4.2. Parameters of six induction machines and loads and their corresponding eigenvalues when operated at 1.0 per unit voltage[a]

Machine	Machine Parameters (per unit)						Load Parameters		Eigen-Values
	R_s	X_s	R_r	X_r	X_m	H	d	β	
1	.013	.13	.015	.09	4.0	.31	.98	2.0	-22.7±j375.5 -13.9±j 48.1 -24.2
2	.014	.10	.018	.08	3.7	.35	.94	2.3	-29.8±j374.0 -20.1±j 49.2 -36.0
3	.016	.13	.020	.08	3.8	.40	.85	1.7	-29.1±j374.2 -19.6±j 41.6 -33.8
4	.020	.20	.009	.07	3.1	.59	.97	1.1	-28.2±j376.1 - 7.3±j 30.4 -11.5
5	.019	.15	.008	.06	3.5	.71	.92	1.8	-34.5±j375.7 - 7.6±j 31.9 -13.9
6	.020	.17	.010	.07	3.0	.63	.88	2.1	-31.8±j375.7 - 8.7±j 31.7 -15.0

[a]The kVA rating of each machine is assumed to be the same. The load parameters a, b and c are 0 for each load.

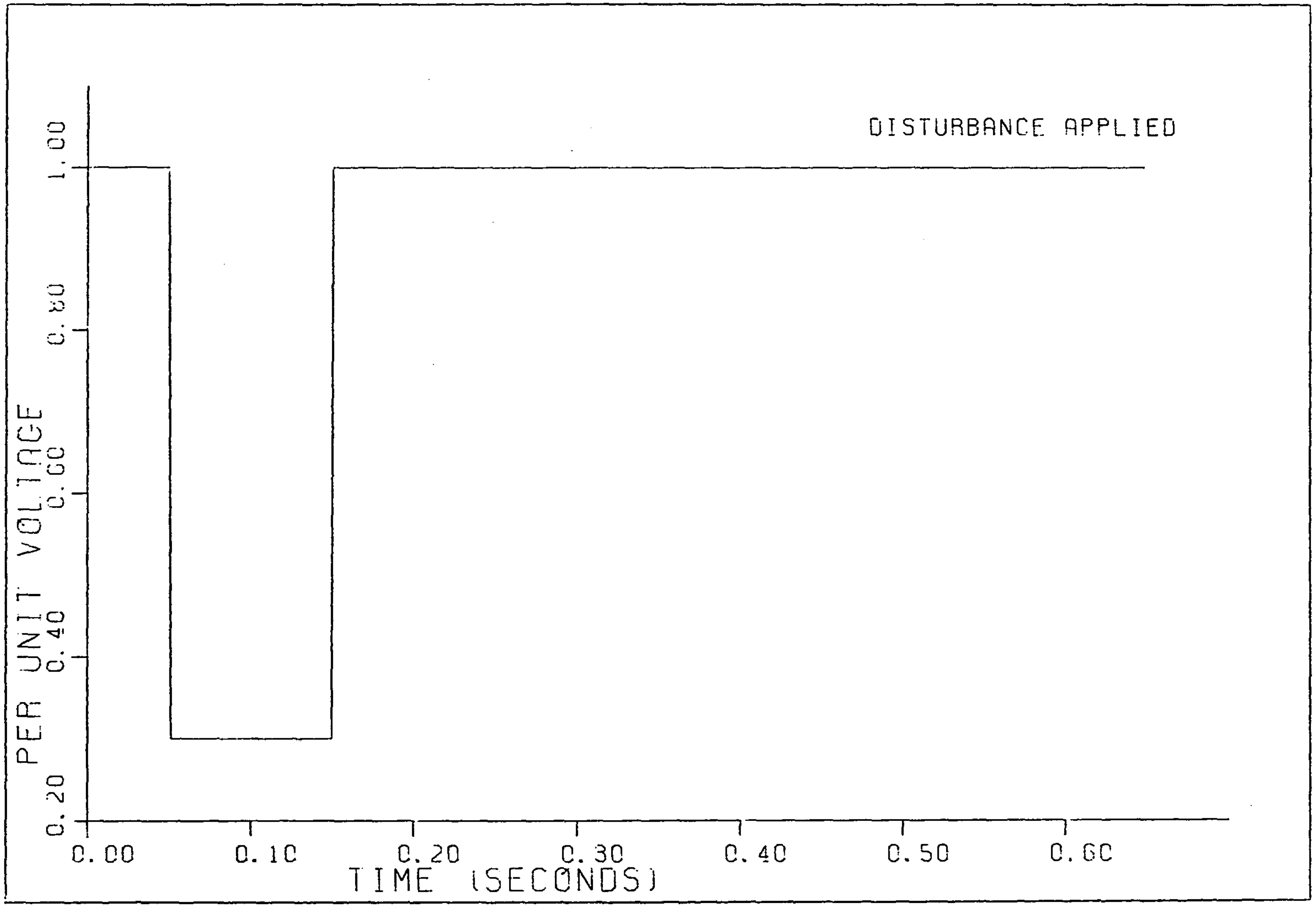

Fig. 4.1. Voltage disturbance applied to the six machines

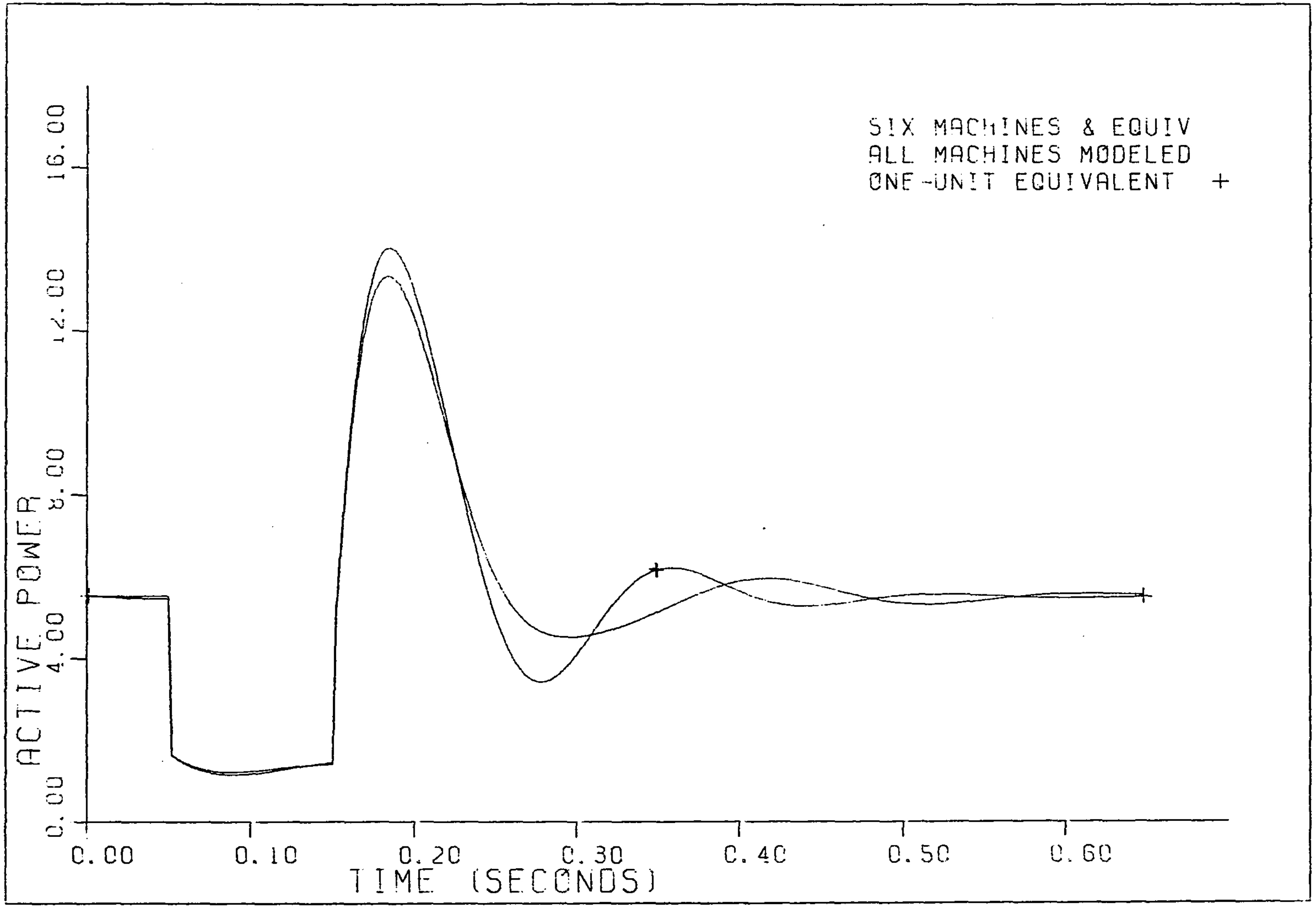

Fig. 4.2. Comparison of the responses of the total average active power drawn by the six machines and by the equivalent machine

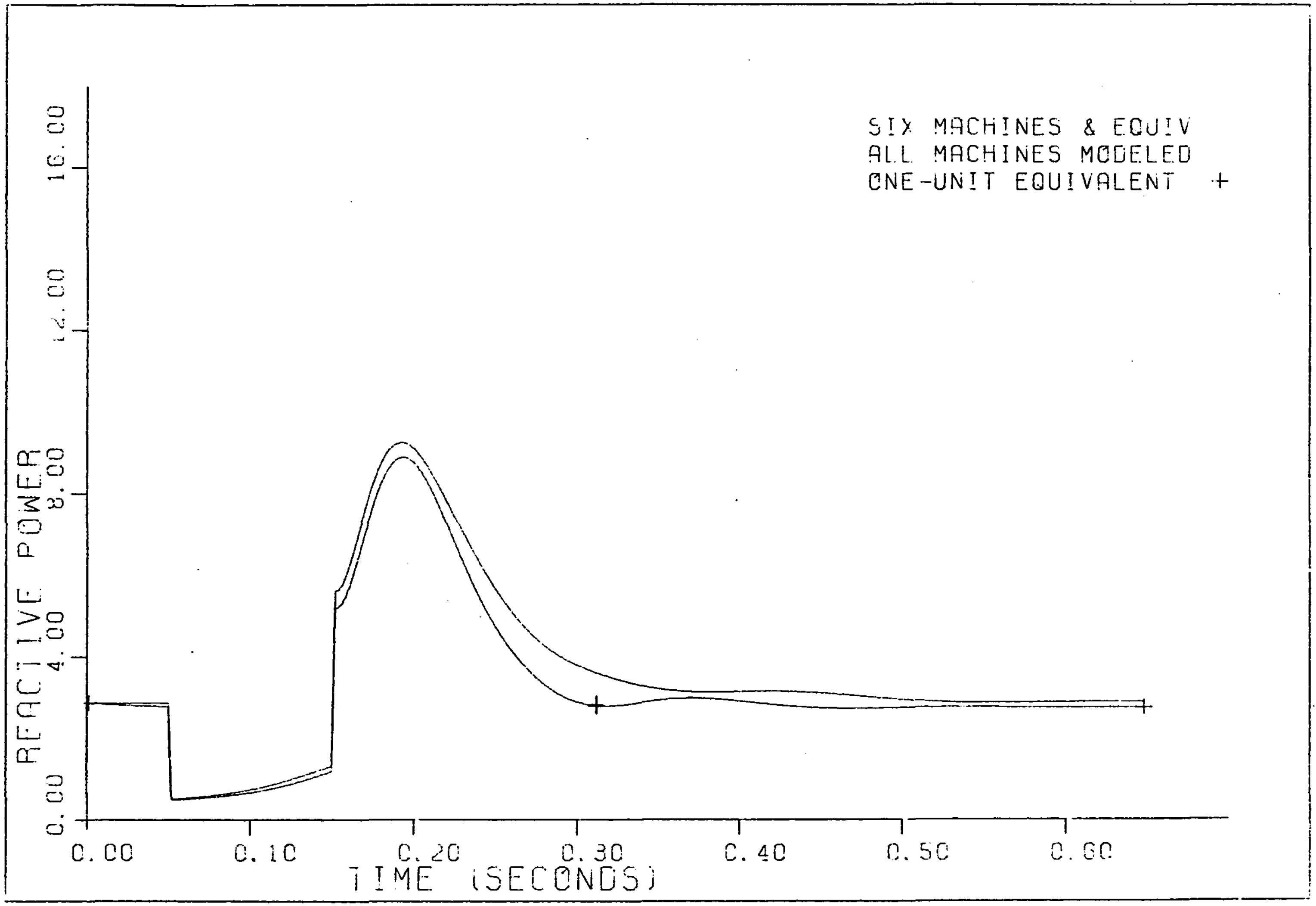

Fig. 4.3. Comparison of the responses of the total average reactive power drawn by the six machines and by the equivalent machine

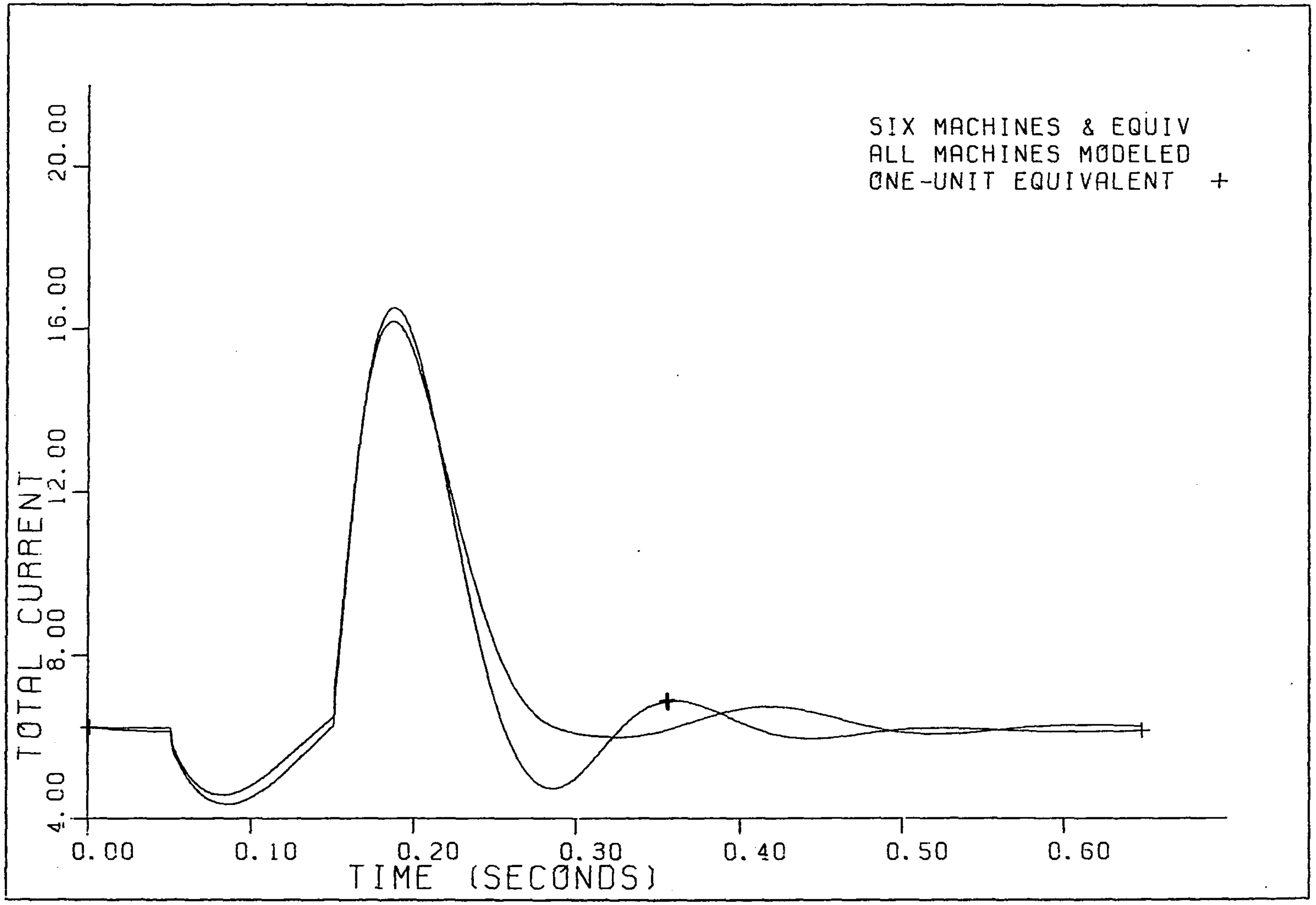

Fig. 4.4, Comparison of the responses of the total average current drawn by the six machines and by the equivalent machine

voltage disturbance is applied in both cases. From this comparison, it is clear that Eq. 4.6 and Eq. 4.7 provide a method to insure that the equivalent machine would give good results during the entire period following the disturbance.

Even though the criteria developed for grouping the machines seem to be rather stringent, only a few equivalent machines would be needed to represent the whole range of induction machines found in power systems. To illustrate this, the eigenvalues of a number of machines of different ratings [28, 38] are computed and listed in Table 4.3. Each machine is assumed to be operating at full load and driving a mechanical load whose inertia is equal to that of the machine.

The eigenvalues listed in Table 4.3 suggest that most small machines (e.g. machines of rating 100 hp and less) may be grouped into one equivalent. The medium size machines (e.g. machines of rating between 100 hp and 2250 hp) would also form another equivalent. The very big machines (e.g. the one rated 6000 hp) may not fit into any group and may have to be modeled separately.

D. Summary

During power system disturbances, energy oscillations occur between the induction machines and the power system. These oscillations have different frequencies for different

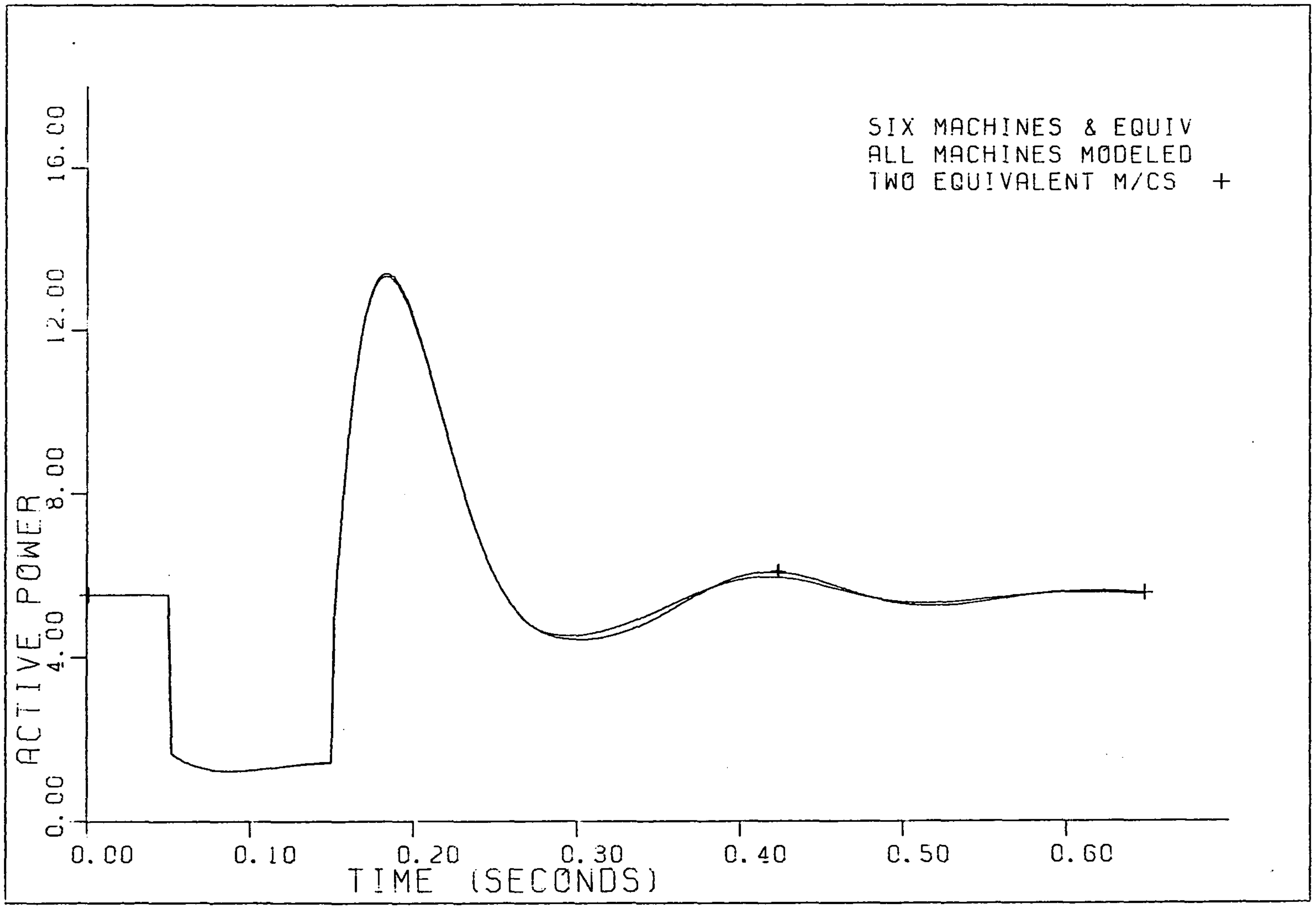

Fig. 4.5. Comparison of the responses of the total average active power drawn by the six machines and by the two equivalent machines

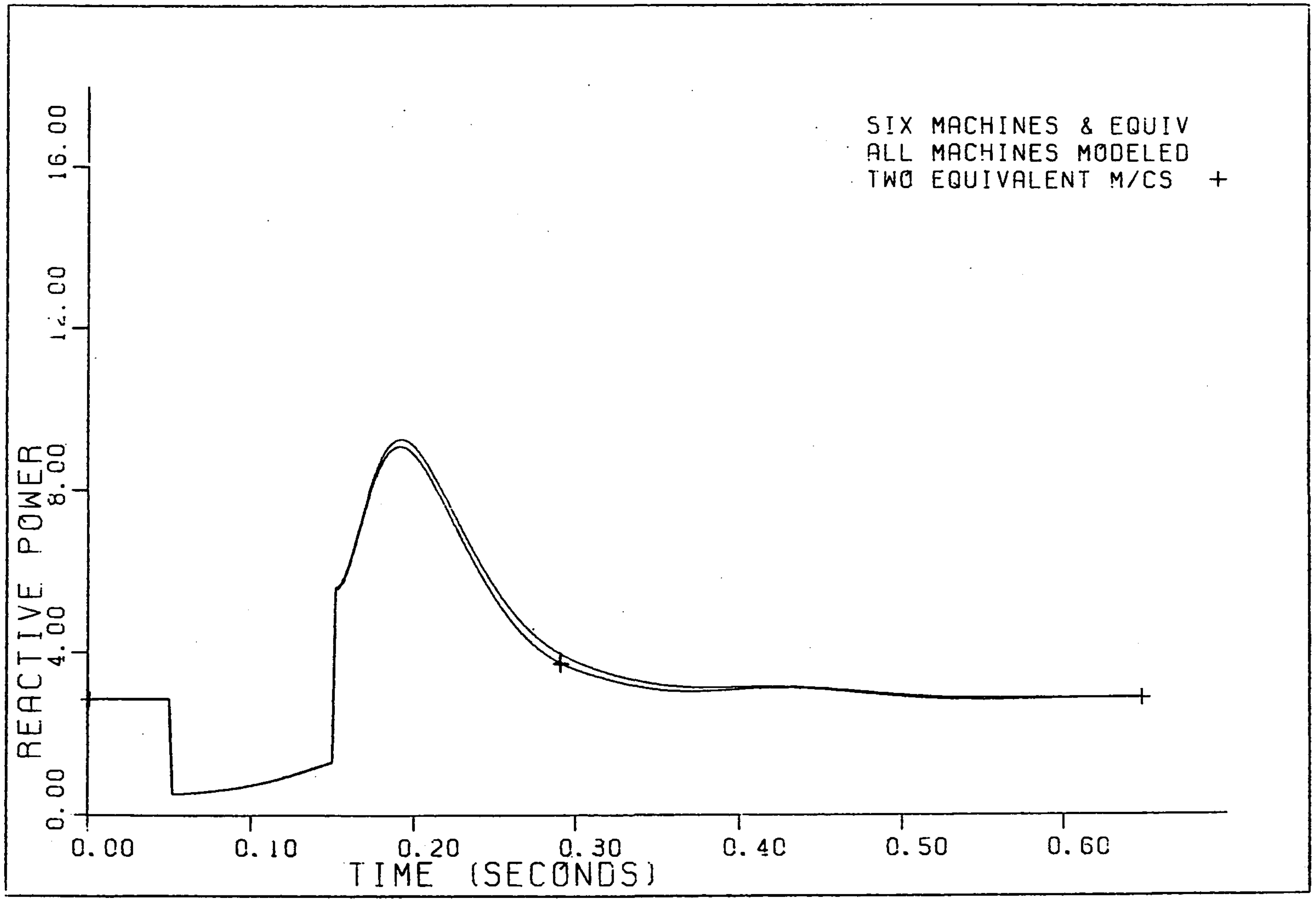

Fig. 4.6. Comparison of the responses of the total average reactive power drawn by the six machines and by the two equivalent machines

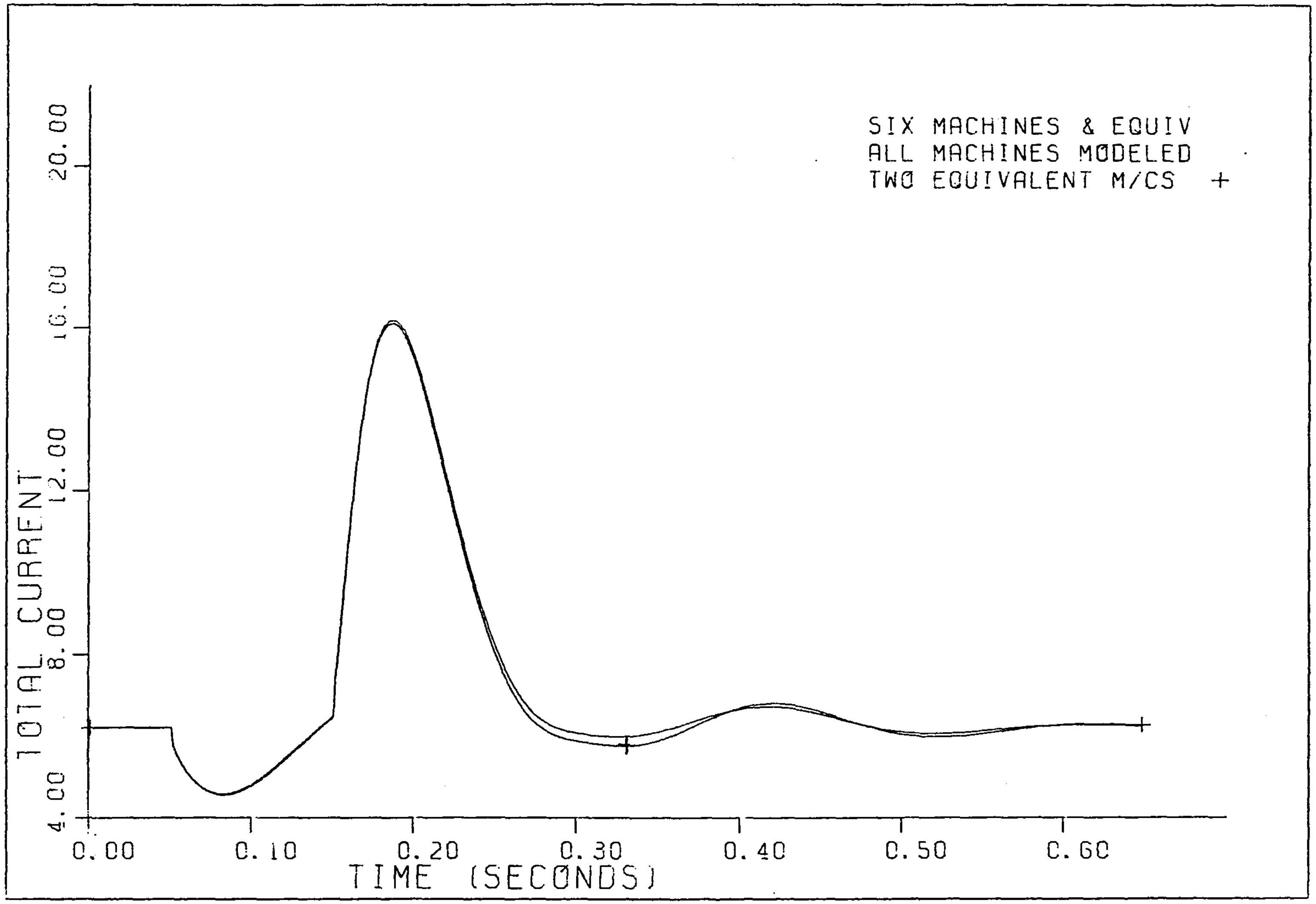

Fig. 4.7. Comparison of the responses of the total average current drawn by the six machines and by the two equivalent machines

Table 4.3. Eigenvalues of typical induction machines

Machine Rating (horsepower)	Eigenvalues Corresponding to the Dynamics
3	-221.8 ± j80.1
25	-185.1 ± j62.5
50	-142.0 ± j38.8
100	-170.0 ± j35.9
250	- 16.9 ± j35.7
500	- 15.6 ± j41.4
800	- 12.9 ± j39.4
1000	- 12.1 ± j36.4
1500	- 9.5 ± j38.2
2250	- 9.4 ± j41.6
6000	- 7.2 ± j19.5

machines. Only machines whose oscillations are almost in phase may be combined into one equivalent. In addition, the duration of the transient phenomena in all machines in one group must not be too different. Satisfactory criteria that machines should meet to be combined into a single equivalent are given in Eq. 4.6 and Eq. 4.7. These criteria limit the error in the grouping to a very small value.

More than one equivalent machine may have to be used to represent a large number of different induction machines in order to achieve good accuracy for the entire period following a disturbance. Using only one equivalent machine to represent a very large group of induction machines with different characteristics would give accurate results only for the first few cycles and the last few cycles of the transient period and thereafter. For the intervening time interval, inaccuracies may arise due to either phase differences in the energy oscillations experienced by the different machines, or due to the different settling times of the transients in the different machines.

V. MODEL APPLICATIONS

A. Introduction

The method developed in Chapter II and Chapter IV for grouping a number of induction machines into equivalent machines could be used to model loads for power system analyses. In any steady state analysis (like loadflows), all the induction machines at any load bus could be grouped into a single-unit equivalent and placed at that bus. This would be accurate because a single-unit equivalent has been found to accurately predict the steady state power drawn by any group of induction machines. For modeling the induction machine loads in the dynamic state (as in a transient stability study), all the induction machine loads could be grouped together into a few equivalent machines at the load buses. Simulating the active and reactive power absorbed by these equivalent machines during transients would require the solution of only a few equations.

When an induction machine is considered as a load in the dynamic state, the average variations in the active and the reactive power are the main quantities of interest. The high frequency (e.g., 60-Hz) electrical transients in these quantities could be suppressed in simulating the machine equations. References 28 and 38 show methods which solve only three differential equations for each induction machine thus supressing the high frequency electrical transients.

Since only one, two or three equivalent machines would be required to represent all induction machines at each load bus, the total number of differential equations that must be solved for the load will not be more than nine. This is comparable to the number of equations used for synchronous generators and their exciters. When generators are modeled in detail, as many as eight differential equations may be required to represent them and their excitation systems. Representing induction machine loads by a few equations would not increase the order of the system of equations in a power system study beyond bounds.

B. An Example of Modeling Load Dynamics in a Transient Stability Study

The method proposed in this dissertation is employed to find equivalent induction machines that are used as loads at three load buses in the test power system shown in Fig. 5.1. The test system is the WSCC 9-bus, three-generator system [4]. The active power consumed by the load at bus 5 is 125 MW. At bus 6 it is 90 MW and at bus 8, the load draws 100 MW. Thirty-one percent (or 39 MW) of the power at bus 5 is assumed to be consumed by induction machines. At bus 6, 41% (or 37 MW) of the load is assumed to be induction machines and at bus 8, it is assumed to be 32% (or 32 MW). The rest of the loads are assumed to be constant impedances.

A number of 30-hp and 100-hp induction machines are grouped together to obtain the equivalent machines needed for the load buses. Enough of these induction machines are used at each load bus to consume the total active power that is assumed to be drawn by induction machine loads. Because the 30-hp and 100-hp machines satisfy the grouping criteria of Chapter IV, only one equivalent machine is computed for each load bus. The parameters of these equivalent machines are listed in Table 5.1.

For the active and reactive power variations at the load buses, a single set of third order differential equations are solved for each equivalent machine. A total of nine differential equations are required for the loads at the three buses. This compares favorably with twenty-four differential equations used for the three generators and their excitation systems.

As a disturbance in the system, a three phase fault is applied at bus 7 for three cycles and then removed. The postfault configuration of the system is the same as the prefault configuration. No lines are switched in clearing the fault.

For comparison, the simulation is performed for three cases. One includes the equivalent machines described

Table 5.1. Parameters of the equivalent induction machines[a]

Parameters	Values (Per Unit on Machine Base)		
	Bus 5	Bus 6	Bus 8
Rating (MVA)	39	37	32
R_s	.0156	.0242	.0189
X_s	.0782	.1212	.0947
R_r	.0094	.0146	.0114
X_r	.1710	.2650	.2070
X_m	3.0400	4.7120	3.6810
H	.4200	.2700	.3470

[a]The load torque on each machine is assumed to be given by $T = 1.0\ \omega^2$.

earlier as part of the load. The others use 100% constant current or 100% constant impedance loads at all load buses. The active and the reactive power variations at bus 5 have been plotted in Fig. 5.2 and Fig. 5.3 for the three cases. The relative rotor angles of the generators in Fig. 5.1 are also plotted for the three cases and are shown in Fig. 5.4.

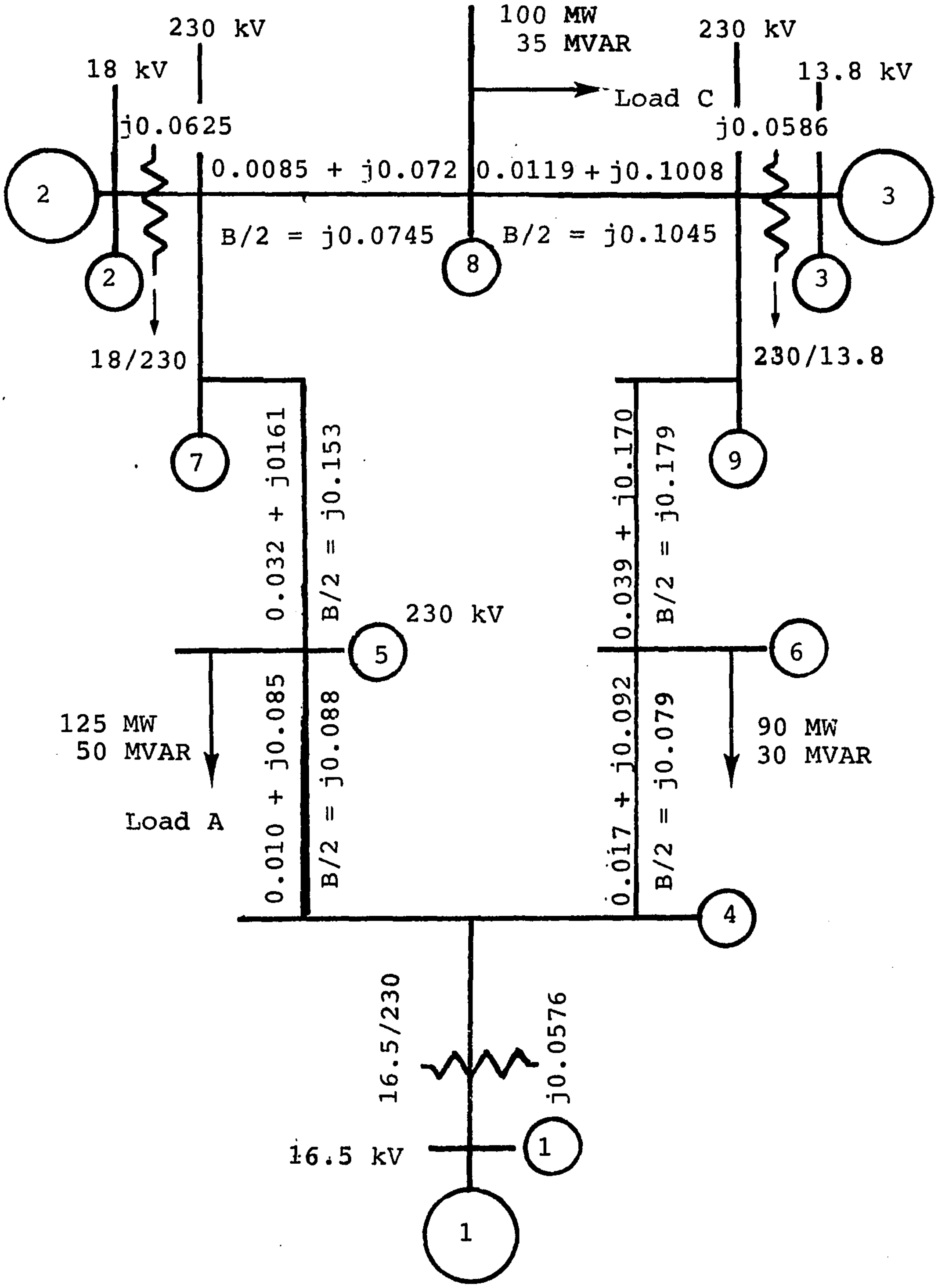

Fig. 5.1. The WSCC 9-bus test system

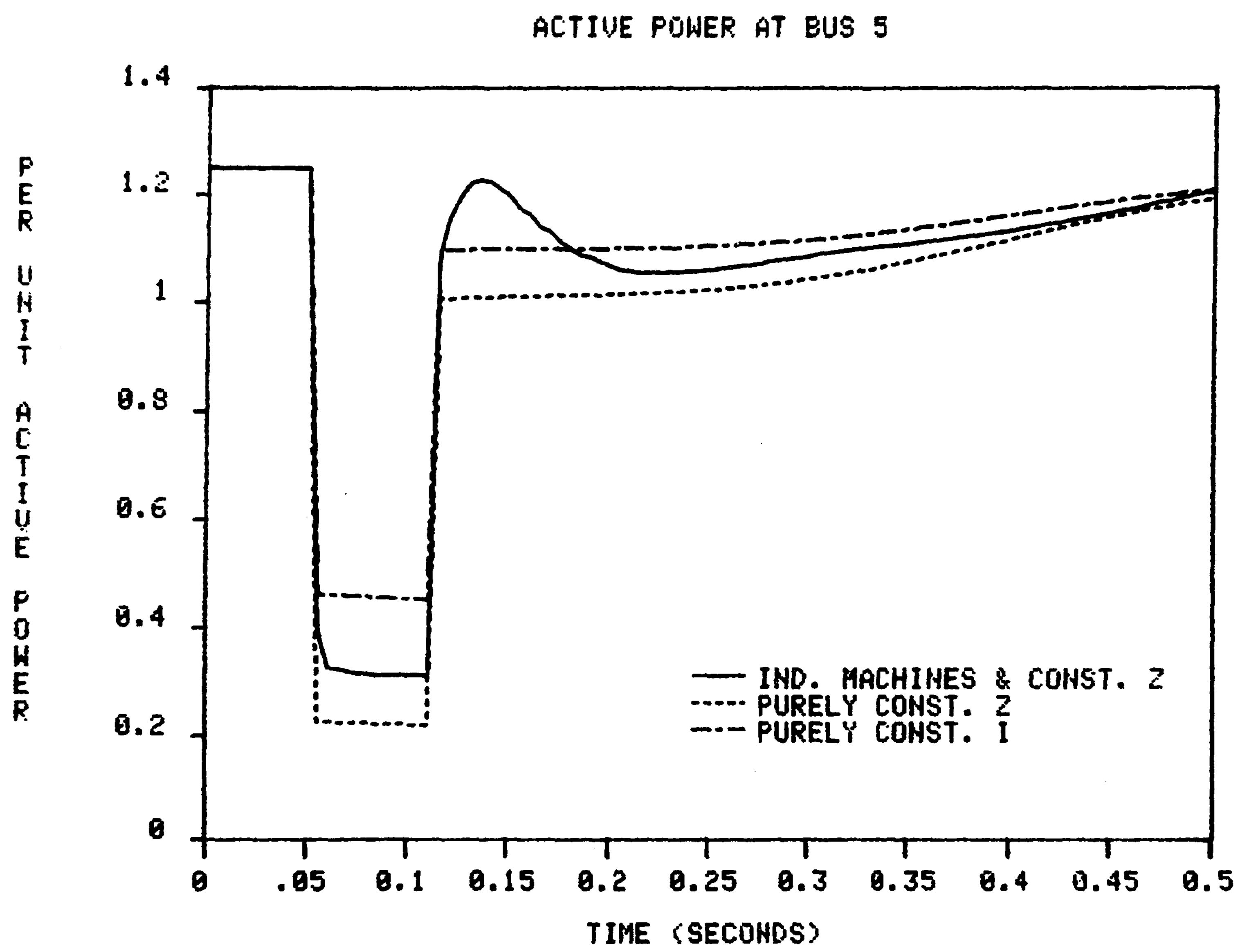

Fig. 5.2. The active power drawn by the load at bus 5

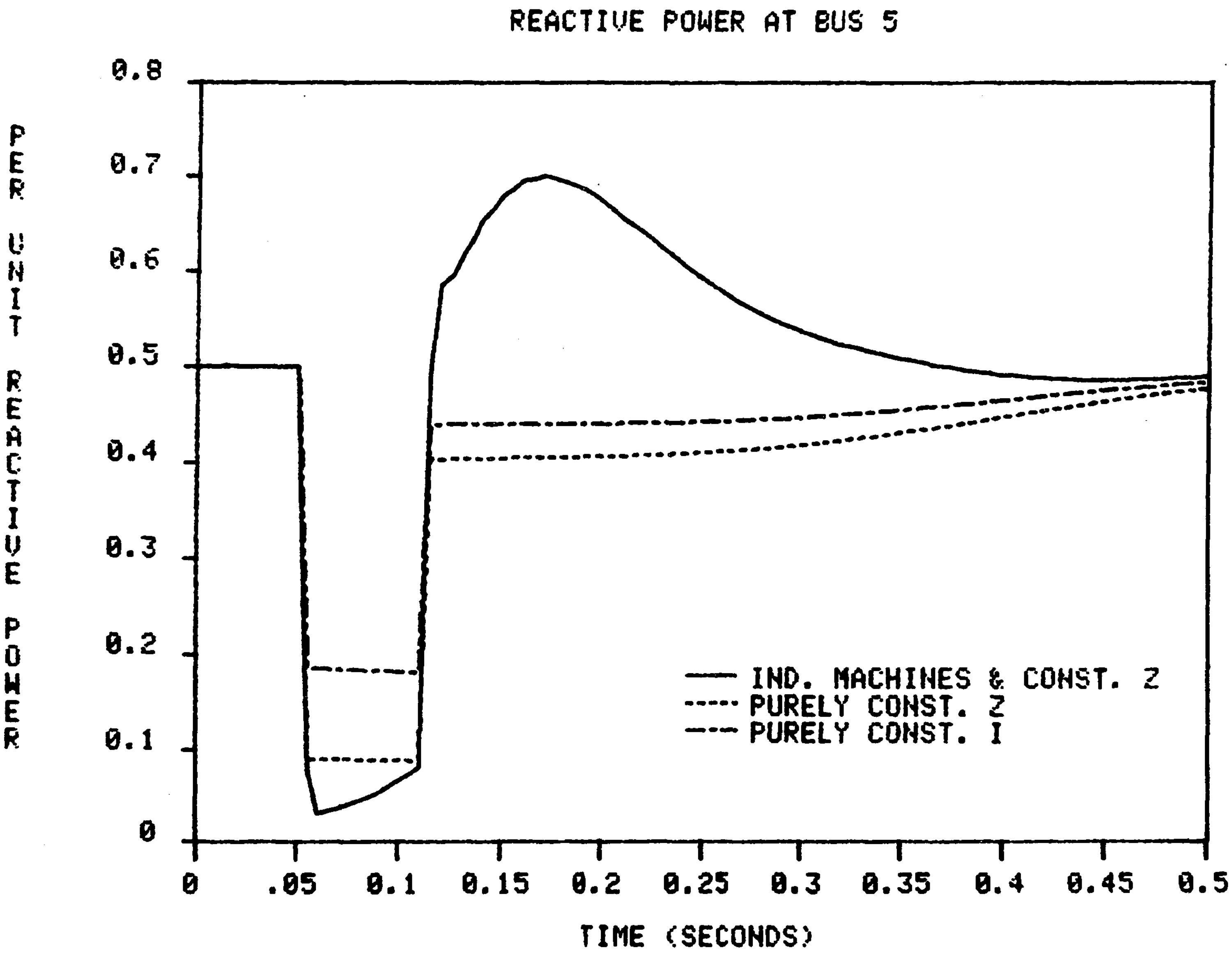

Fig. 5.3. The reactive power drawn by the load at bus 5

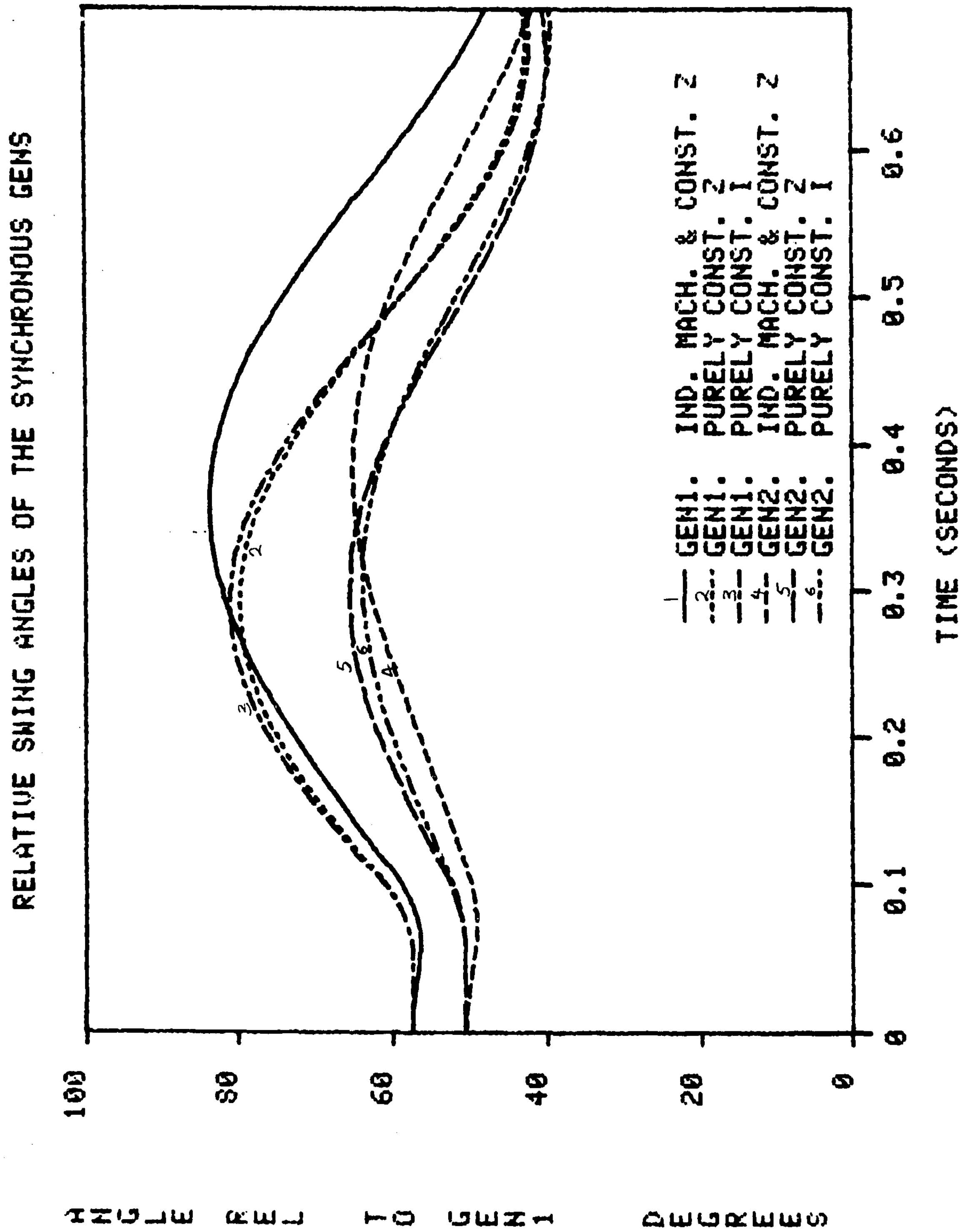

Fig. 5.4. The relative rotor angles of the synchronous generators with respect to generator 1

C. Analysis of Simulation Results

From the results of the transient stability study, the load shows significant dynamics following the disturbance when induction machines are included in the load model. This may be seen in Fig. 5.2 and Fig. 5.3, where both the active and the reactive power show peaks that correspond to the dynamics in these quantities. The constant current or constant impedance load models do not show the same dynamic behavior. As may be observed from Fig. 5.4, modeling the load dynamics gives relative swing angles of the generators that are different from those obtained by using static load models.

Since it has been determined [14, 22] in practice that loads exhibit dynamic behavior especially following power system disturbances, it would be appropriate to represent the dynamics in any power system study where the response of the load is important. This means that induction machines, which make up more than half of most loads, should be represented in the load model as machines and not as static loads. The method proposed and illustrated in this dissertation would do this.

D. Summary

The application of the method of grouping induction machines to obtain an equivalent induction machine for modeling part of the load has been demonstrated. The equivalents determined for three groups of induction machines are used in a transient stability study to demonstrate the dynamic behavior of the load. When induction machines are modeled, the loads showed significant dynamics following disturbances.

Practical measurements [14, 22] reveal that power system loads exhibit significant dynamics. It may be important to model these dynamics in studies where knowledge of the load behavior during the transient period is required or where the load model affects the results significantly.

Only three differential equations need to be solved to obtain the average power variations to each equivalent induction machine load during power system disturbances. This is advantageous especially if more than one equivalent induction machine must be used at a load bus. However, not more than three equivalent machines may be required to represent the induction machines at most load buses in power systems. Therefore, using this method will not cause the number of equations to be solved in any study to increase beyond bounds.

VI. CONCLUSIONS

A. Summary of Research Findings

Some analyses of electric power systems require knowledge of the behavior of the loads in the system. Therefore, different load models have been proposed and used for power system studies. In some studies, the results obtained are very sensitive to the model used for the load. In order to reduce the uncertainties in the results of such studies, accurate load models have been sought.

The most widely used models for power system loads are the constant current, constant impedance and constant MVA representations and various combinations thereof. All these are classified as static models because they do not include any inertial effects. Static models do not, in general, predict the oscillations of energy that are usually observed between the load and the power system following system or load disturbances. These oscillations of energy, or the dynamics as they are called, are mainly due to the induction machines, which make up more than 60% of most loads. The static load models are therefore not adequate for modeling these effects.

The induction machines encountered in power systems are nonhomogeneous. They have different ratings and characteristics and they drive different types of loads. It is impractical to use a full representation of each individual induction machine in studies involving more than a few machines. Therefore, it is desirable to find methods that do not require full representation of all the individual induction machines in the power system. This dissertation has suggested a method by which this could be done.

The proposed method computes the parameters of an equivalent induction machine that would represent a large number of induction machines (and their feeders) connected to a common bus. The active power drawn by the equivalent machine equals the sum of the active power drawn by all the machines it replaces. Similarly, the reactive power (or the current) drawn by the equivalent machine equals the total reactive power (or the total current) drawn by the individual machines. The equivalent machine also delivers power to an equivalent mechanical load that equals the sum of the power delivered by all the individual machines to their mechanical loads.

It is possible to combine all the induction machines at any load bus into a single induction machine. However, such a machine may not accurately predict the net behavior of a diversified assortment of induction machines during transients. Following a disturbance at a load bus, a one-unit equivalent machine has been found to adequately describe the net behavior of all machines only at the start of the dynamics and in the steady state. In the period between the onset of the dynamics and the steady state, a one-unit equivalent deviates from predicting the aggregate behavior of some combinations of induction machines connected to the same bus. This is especially so when machines that respond very differently to system disturbances are grouped into a single unit.

The frequency of the dynamics experienced by induction machines vary from one machine to another. They depend especially on the machine size, and the its total inertia. Similarly, the duration of the dynamics also depend on the machine size and parameters. It has been found that, due to differences in the frequency and the duration of the dynamics, lumping all induction machines into one equivalent may not give results that match the actual results obtained from modeling all machines individually. Two criteria for deciding which machines could be grouped together to achieve acceptable error levels in the equivalent during the transient period have been developed.

An example that uses the proposed grouping method to represent a large number of induction machines by an equivalent induction machine at a load bus for a transient stability study has also been included in this research. The example is done on the WSCC 9-bus power system. The results of the example clearly demonstrate the dynamic behavior of the loads following power system disturbances. No dynamic behavior is observed when constant current or constant impedance loads are used in a similar example. Only a single set of third order differential equations need to be solved for each equivalent machine to obtain the average power variations to the induction machine loads during power system disturbances. This is advantageous especially when more than one equivalent induction machine must be used at a load bus.

The method developed in this research requires knowledge about the induction machines to be grouped together. In practice, the parameters and operating conditions of all machines at any load bus may not be known. A statistical method for obtaining equivalent induction machines at load buses in practical power systems is suggested in the following section.

B. Suggestions for Obtaining Equivalent Induction Machines for Practical Power System Load Buses

The method presented so far assumes that the parameters of the induction machines, feeders, and the mechanical loads at all load buses are known. In practice, these parameters are not known. It may, therefore, be necessary to use some estimated values for the induction machine and load parameters. The following procedure for obtaining estimates of these parameters is being suggested for practical power systems.

1. The range of induction machines commonly found in power systems should be divided into classes based on their ratings. Representative machines and mechanical loads should be determined for each class. The determination of the parameters could be done theoretically by grouping large numbers of different machines in each class, including their feeders and mechanical loads, into equivalents and scaling down the rating of the equivalent to the rating of the class being considered. These representative induction machines would fairly accurately describe the aggregate behavior of a large number of induction machines in that particular class.
2. By using load surveys or market data on the number of equipment sold that have induction machines, an estimate of the number of induction machines (and their classes) in a residential or commercial load area could be obtained. As an example, consider one such type of equipment, say, washers. The number of washers in the load area could be estimated from the load survey. The utilization factors for washers at any time

of the day should be known or determined. Multiplying the estimated number of washers and the utilization factors would give an estimate of the actual number of washers in use at a particular time. The class of machines in washers should be known or determined. The parameters of the representative machine in this class may be used for the machine in each washer. The equivalent for all the induction machines in the washers could then be found. This procedure may be repeated for other types of equipment and their corresponding equivalent induction machines also found. The equivalent machines for all the different types of equipment connected to the bus may then be grouped together into a few equivalents at the load bus.

3. For industrial areas, where there are bigger and more specialized induction machines, the total power consumed by all the induction machine loads may be estimated as a fraction of the total power drawn by all loads in the area. The power drawn by the different classes of machines at any time of the day should be determined from a load survey. This should be expressed as a percentage of the total induction machine load. At any

other time, the power drawn by the induction machines in any particular class could then be estimated from the percentage determined for it. A number of representative induction machines of this class may be used together at the bus to consume the same amount of power that is estimated for this class. This procedure should be carried out for all the classes of machines present. The equivalent machines for all the classes may then be grouped into a few equivalent machines at the load bus using the grouping criteria.

If there are any special machines (e.g., very large machines) at a load bus whose parameters and operating conditions are known, they may be treated separately. The active power drawn by such machines should be deducted from the total active power drawn by all induction machines at the load bus before the equivalent machines are determined. If these special machines are compatible with any other class of machines at the bus, they may be grouped together into the equivalent of that class. Otherwise, they should be modeled separately.

VII. BIBLIOGRAPHY

1. Abdel-Hakim, M. M., and G. J. Berg. "Dynamic Single-Unit Representation of Induction Motor Groups." IEEE Transactions PAS-95 (1976): 155-165.

2. Adler, R. B., and C. C. Mosher. "Steady-State Voltage Power Characteristics for Power System Loads." Paper 70 CP 706-PWR. IEEE Summer Power Meeting. New York: IEEE, 1970.

3. Akhtar, M. Y. "Frequency-Dependent Dynamic Representation of Induction-Motor Loads." Proceedings IEE 115 (June 1968): 802-812.

4. Anderson, P. M., and A. A. Fouad Power System Control and Stability. Vol. 1. Ames, Iowa: Iowa State University Press, 1977.

5. Berg, G. J. "System and Load Behavior Following Loss of Generation. Experimental Results and Evaluation." Proceedings IEE 119 (1972): 1483-1486.

6. Berg, G. J. "Power System Load Representation." Proceedings IEE 230 (1973): 344-348.

7. Berg, G. J., and P. Subramaniam. "Induction Motor Load Representation." Paper A79 472-2. IEEE PES Summer Meeting, Vancouver. New York: IEEE, 1979.

8. Brereton, D. S., D. G. Lewis, and C. C. Young. "Representation of Induction-Motor Loads During Power System Stability Studies." Trans. AIEE 76 (1957): 451-460.

9. Cathey, J. J., R. K. Calvin III, and A. K. Ayoub. "Transient Load Model of an Induction Motor." IEEE Transactions PAS-92 (1973): 1399-1406.

10. Chen, M. S. Determining Load Characteristics for Transient Performance. EPRI Report EL-849, Vol. 1, Project RP849-3. EPRI, Palo Alto, CA, 1979.

11. Chen, M. S. Determining Load Characteristics for Transient Performance. EPRI Report EL-849, Vol. 2, Project RP849-3. EPRI, Palo Alto, CA, 1979.

12. Chen, M. S. Determining Load Characteristics for Transient Performance. EPRI Report EL-849, Vol. 3. Project RP849-3. EPRI, Palo Alto, CA, 1979.

13. Concordia, C., and S. Ihara. " Load Representation in Power System Stability Studies." IEEE Transactions PAS-101 (1982): 969-977.

14. Gentile, T., S. Ihara, A. Murdoch, and N. Simons. Determining Load Characteristics for Transient Performance. EPRI Report EL-850, Vol. 1, Project 849-1. EPRI, Palo Alto, CA, 1981.

15. Gentile, T., S. Ihara, A. Murdoch, and N. Simons. Determining Load Characteristics for Transient Performance. EPRI Report EL-850, Vol. 2, Project 849-1. EPRI, Palo Alto, CA, 1981.

16. Gentile, T., S. Ihara, A. Murdoch, and N. Simons. Determining Load Characteristics for Transient Performance. EPRI Report EL-850, Vol. 3, Project 849-1. EPRI, Palo Alto, CA, 1981.

17. Gentile, T., S. Ihara, A. Murdoch, and N. Simons. Determining Load Characteristics for Transient Performance. EPRI Report EL-850, Vol. 4, Project 849-1. EPRI, Palo Alto, CA, 1981.

18. Gevay, B. J., and W. H. Schippel. "Transient Stability of an Isolated Radial Power Network with Varied Load Division." IEEE Transactions PAS-83 (1964): 964-970.

19. IEEE CAPS Subcommittee. "System Load Dynamics-Simulation Effects and Determination of Load Constants." IEEE Transactions PAS-92 (1973): 600-609.

20. Ihara, S., and R. Baheti. "A Dynamic Model of Aggregate Induction Motor Load for Large Voltage Dips." Proceedings of 1980 Joint Automatic Control Conference. Vol. II. Paper No. FA3-E. New York: IEEE,1980.

21. Iliceto, F., and A. Capasso. "Dynamic Equivalents of Asynchronous Motor Loads in System Stability Studies." IEEE Transactions PAS-93 (1974): 1650-1659.

22. Iliceto, F., A. Ceyhan, and G. Ruckstuhl. "Behavior of Loads During Voltage Dips Encountered in Stability Studies. Field and Laboratory Tests." IEEE Transactions PAS-91 (1972): 2470-2479.

23. Jordan, H. E. "Analysis of Induction Machines in Dynamic Systems." IEEE Transactions PAS-84 (1965): 1080-1088.

24. Kalsi, S. S., and B. Adkins. "Transient Stability of Power Systems Containing both Synchronous and Induction Machines." Proceedings IEE 118 (1971): 1467-1474.

25. Kent, M. H., W. R. Schmus, F. A. McCrackin, and L. M. Wheeler. "Dynamic Modeling of Loads in Stability Studies." IEEE Transactions PAS-88 (1969): 756-763.

26. Krause, P. C. "Method of Multiple Reference Frames Applied to the Analysis of Symmetrical Induction Machinery." IEEE Transactions PAS-87 (1968): 218-227.

27. Krause, P. C., and J. R. Hake. "Method of Multiple Reference Frames Applied to the Analysis of a Rectifier-Inverter Induction Motor Drive." IEEE Transactions PAS-88 (1969): 1635-1641.

28. Krause, P. C., and A. Murdoch III. "Simplified Representation of Induction Machine Dynamics." Paper C75 132-6. IEEE PES Power Meeting, New York. New York: IEEE, 1975.

29. Krause, P. C., and C. H. Thomas. "Simulation of Symmetrical Induction Machinery." IEEE Transactions PAS-84 (1965): 1038-1053.

30. Krause, P. C., F. Nozari, T. L. Skvarenina, and D. W. Olive. "The Theory of Neglecting Stator Transients." IEEE Transactions PAS-98 (1979): 141-148.

31. Lipo, T. A., P. C. Krause, and H. E. Jordan. "Harmonic Torque and Speed Pulsations in a Rectifier-Inverter Induction Motor Drive." IEEE Transactions PAS-88 (1969): 579-587.

32. Magginnis, F. J., and N. R. Schultz. "Transient Performance of Induction Motors." Trans. AIEE 63 (1964): 641-646.

33. Mahmoud, A. A., R. G. Harley, and C. Calabrese. "Induction Motor Transient Performance Prediction in Distribution Network." Paper No. A78-536-5. IEEE PES Summer Power Meeting. New York: IEEE, 1978.

34. Mauricio, W., and A. Semlyen. "Effect of Load Characteristics on the Dynamic Stability of Power Systems." IEEE Transactions PAS-92 (1972): 2295-2304.

35. Ramsden, V. S., N. Zorbas, and R. R. Booth. "Prediction of Induction Motor Dynamic Performance in Power Systems." Proceedings IEE 115 (1968): 511-518.

36. Robertson, S. D. T., and K. M. Hebbar. "A Digital Model for Three-Phase Induction Machines." IEEE Transactions PAS-88 (1969): 1624-1634.

37. Roohparvar, F., A. A. Mahmoud, and J. I. Hanania. "Multi-Machine Modeling of a Group of Three-Phase Induction Motors." Proceedings ICEE April 1981: 24-31.

38. Sastry, K. P. R., and R. E. Burridge. "Investigation of Reduced Order Model for Induction Machine Dynamic Studies." IEEE Transactions PAS-95 (1976): 962-969.

39. Siskind, C. S. Electrical Machines, Direct and Alternating Current. New York, NY: McGraw-Hill Book Company, Inc., 1959.

40. Skvarenina, T. L., and P. C. Krause. "Accuracy of a Reduced Order Model Induction Machines in Dynamic Stability Studies." IEEE Transactions PAS-95 (1979): 1192-1197.

41. Stanley, H. C. "An Analysis of the Induction Machine." Trans. AIEE 57 (1938): 751-755.

42. U.S. Department of Energy. Energy Efficiency of Electric Motor. Report No. HCP/M5017-01. Washington D.C.: U.S. DOE, 1980.

43. Ueda, R., and S. Takata. "Effects of Induction Machine Load on Power System." IEEE Transactions PAS-100 (1981): 2555-2562.

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IX. APPENDIX A: THE EQUATIONS OF A SYMMETRICAL INDUCTION MACHINE

To simplify the derivation of the induction machine equations, it is assumed that:

1. The magnetic circuits of both the rotor and stator are linear. Hysteresis and eddy currents are neglected.
2. The induction machine is radially symmetrical, the self-inductance of the windings are not dependent on the rotor angular position. All stator phase circuits are identical. The rotor phase circuits are also identical.
3. The applied voltage is balanced and sinusoidal.

The voltage on an induction machine is related to the currents in the machine by Eq. 9.1.

$$[v]_{abc} = [R]\,[i]_{abc} + p\{[\ell]\,[i]_{abc}\} \qquad (9.1a)$$

where

$$[v]_{abc} = [v_{as} \quad v_{bs} \quad v_{cs} \quad v_{ar} \quad v_{br} \quad v_{cr}]^T \qquad (9.1a)$$

$$[i]_{abc} = [i_{as} \quad i_{bs} \quad i_{cs} \quad i_{ar} \quad i_{br} \quad i_{cr}]^T \qquad (9.1b)$$

$$[R] = \begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix} = \mathrm{diag}(R_s \quad R_s \quad R_s \quad R_r \quad R_r \quad R_r) \qquad (9.1c)$$

R_s = resistance of stator circuit

R_r = resistance of rotor circuit

The subscripts a, b and c refer to the phases, subscript s refers to the stator, and subscript r refers to the rotor.

$$[\ell] = \begin{bmatrix} L_{ss} & L_{sr} \\ L_{sr} & L_{rr} \end{bmatrix} \qquad (9.1d)$$

$$= \begin{bmatrix} L_{aas} & L_{abs} & L_{acs} & L_{as-ar} & L_{as-br} & L_{cs-cr} \\ L_{bas} & L_{bbs} & L_{bcs} & L_{bs-ar} & L_{bs-br} & L_{bs-cr} \\ L_{cas} & L_{cbs} & L_{ccs} & L_{cs-ar} & L_{cs-br} & L_{cs-cr} \\ L_{ar-as} & L_{ar-bs} & L_{ar-cs} & L_{aar} & L_{abr} & L_{acr} \\ L_{br-as} & L_{br-bs} & L_{br-cs} & L_{bar} & L_{bbr} & L_{bcr} \\ L_{cr-as} & L_{cr-bs} & L_{cr-cs} & L_{car} & L_{cbr} & L_{ccr} \end{bmatrix}$$

(9.1e)

For symmetrical machines,

$$L_{aar} = L_{bbs} = L_{ccs} = L_s$$
$$L_{aar} = L_{bbs} = L_{ccr} = L_r$$
$$L_{abs} = L_{acs} = L_{bas} = L_{bcs} = L_{cas} = L_{cbs} = L_{sm}$$
$$L_{abr} = L_{acr} = L_{bar} = L_{bcr} = L_{car} = L_{cbr} = L_{mr}$$
$$L_{as-ar} = L_{br-br} = L_{cs-cr} = L_{sr}\cos(\theta r)$$
$$L_{as-cr} = L_{bs-ar} = L_{cs-br} = L_{sr}\cos(\theta r-2\pi/3)$$
$$L_{as-br} = L_{bs-cr} = L_{cs-ar} = L_{sr}\cos(\theta r+2\pi/3) \qquad (9.1f)$$

where

θr is defined in Eq. 9.2f and figure 9.1

L_{aas} is the self-inductance of stator phase winding

L_{aar} is the self-inductance of rotor phase circuit

L_{sm} is the mutual inductance between stator phase windings

L_{mr} is the mutual inductance between rotor phase circuits

L_{sr} is the mutual inductance between rotor and stator circuits

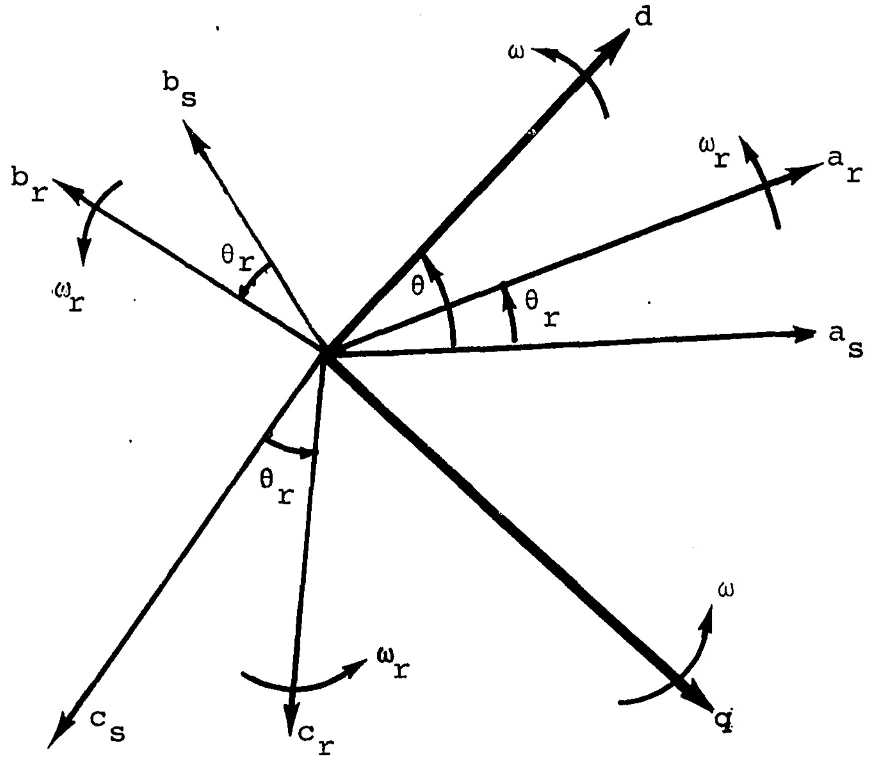


Figure 9.1 The relationship between the axes of the induction machine

Equations 9.1 are in the a-b-c phase variables. These can be greatly simplified if they are transformed into the zero, direct and quadrature axes by the well-known Park's transformation. Eq. 9.2a shows the transformation.

$$[P]\ [v]_{abc} = [P]\ [R]\ [i]_{abc} + [P]\ p\{[\ell]\ [i]_{abc}\} \qquad (9.2a)$$

where

$$p = d/dt \qquad (9.2b)$$

$$[P] = \begin{bmatrix} P\theta & 0 \\ 0 & P\beta \end{bmatrix} \qquad (9.2c)$$

$$[P\theta] = \sqrt{2/3} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3) \\ \sin\theta & \sin(\theta-2\pi/3) & \sin(\theta+2\pi/3) \end{bmatrix} \qquad (9.2d)$$

$$[P\beta] = \sqrt{2/3} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\beta & \cos(\beta-2\pi/3) & \cos(\beta+2\pi/3) \\ \sin\beta & \sin(\beta-2\pi/3) & \sin(\beta+2\pi/3) \end{bmatrix} \qquad (9.2e)$$

$\theta = \theta r + \beta$

$\theta r = \omega_r t$

$\theta = \omega t$

ω = speed of reference frame

ω_r = speed of rotor (9.2f)

In Eq. 9.2a, let

$$[\ell]\ [i]_{abc} = [\lambda]_{abc} \qquad (9.3a)$$

$$[\lambda]_{0dq} = [P]\ [\lambda]_{abc} = [P]\ [\ell]\ [i]_{abc} \qquad (9.3b)$$

$$[v]_{0dq} = [P]\ [v]_{abc} \tag{9.3c}$$

$$[i]_{0dq} = [P]\ [i]_{abc} \tag{9.3d}$$

Therefore,

$$[P]\ [R]\ [i]_{abc} = [P]\ [R]\ [P]^{-1}\ [i]_{0dq} = [R]\ [i]_{0dq} \tag{9.3e}$$

$$[P]\ p\{[\ell]\ [i]_{abc}\} = [P]\ p\{[\lambda]_{abc}\} \tag{9.3f}$$

From Eq. 9.3b,

$$p[\lambda]_{0dq} = p\{[P]\}\ [\lambda]_{abc} + [P]\ p\{[\lambda]_{abc}\} \tag{9.3g}$$

Therefore,

$$[P]\ p[\lambda]_{abc} = p[\lambda]_{0dq} - p\{[P]\}\ [P]^{-1}\ [\lambda]_{0dq} \tag{9.3h}$$

$$= p[\lambda]_{0dq} + \begin{bmatrix} 0 \\ \omega\lambda_{qs} \\ -\omega\lambda_{ds} \\ 0 \\ (\omega-\omega_r)\lambda_{qr} \\ -(\omega-\omega_r)\lambda_{dr} \end{bmatrix} \tag{9.3i}$$

$$= p[\lambda]_{0dq} + [V\omega]_{0dq} \tag{9.3j}$$

where

$$\omega = p\theta$$

$$\omega-\omega_r = p\beta \tag{9.3k}$$

$$[V\omega]_{0dq} = [0 \quad \omega\lambda_{qs} \quad -\omega\lambda_{ds} \quad 0 \quad (\omega-\omega_r)\lambda_{qr} \quad -(\omega-\omega_r)\lambda_{dr}]^T \tag{9.3l}$$

Also, from Eq. 9.3b,

$$[\lambda]_{0dq} = [P]\ [\ell]\ [P]^{-1}\ [i]_{0dq} = [L_{0dq}]\ [i]_{0dq} \tag{9.3m}$$

$$[L_{0dq}] = [P]\ [\ell]\ [P]^{-1}$$

$$= \begin{bmatrix} L_s-2L_{sm} & 0 & 0 & 0 & 0 & 0 \\ 0 & L_s+L_{sm} & 0 & 0 & 3/2L_{sr} & 0 \\ 0 & 0 & L_s+L_{sm} & 0 & 0 & 3/2L_{sr} \\ 0 & 0 & 0 & L_r-2L_{mr} & 0 & 0 \\ 0 & 3/2L_{sr} & 0 & 0 & L_r+L_{mr} & 0 \\ 0 & 0 & 3/2L_{sr} & 0 & 0 & L_r/L_{mr} \end{bmatrix} \tag{9.3n}$$

Substituting Eq. 9.3 in Eq. 9.2 gives

$$\begin{bmatrix} v_{0s} \\ v_{ds} \\ v_{qs} \\ v_{0r} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{bmatrix} R_s+L_{0s}p & 0 & 0 & 0 & 0 & 0 \\ 0 & R_s+L_{ss}p & \omega L_{ss} & 0 & Mp & \omega M \\ 0 & -\omega L_{ss} & R_s+L_{ss}p & 0 & -\omega M & -Mp \\ 0 & 0 & 0 & R_r+L_{0r}p & 0 & 0 \\ 0 & Mp & (\omega-\omega_r)M & 0 & R_r+L_{rr}p & (\omega-\omega_r)L_{rr} \\ 0 & -(\omega-\omega_r)M & Mp & 0 & -(\omega-\omega_r)L_{rr} & R_r+L_{rr}p \end{bmatrix} \begin{bmatrix} i_{0s} \\ i_{ds} \\ i_{qs} \\ i_{0r} \\ i_{dr} \\ i_{qr} \end{bmatrix} \tag{9.3o}$$

Equation 9.3o may be written as Eq. 9.4.

$$[v]_{0dq} = [R]\ [i]_{0dq} + [L_{0dq}]\ p[i]_{0dq} + [V\omega]_{0dq} \tag{9.4}$$

From Eq. 9.4,

$$p[i]_{0dq} = [L_{0dq}]^{-1}\ [v]_{0dq} + [L_{0dq}]^{-1}\ \{-[R][i]_{0dq} - [V\omega]_{0dq}\} \tag{9.5}$$

Substituting the expressions for $[V\omega]_{0dq}$ from Eq. 9.31 into Eq. 9.5 gives

$$p[i]_{0dq} = [D_f]\,[v]_{0dq} + [E_f]\,[i]_{0dq} \tag{9.6a}$$

where

$$[D_f] = [L]^{-1}_{0dq} \tag{9.6b}$$

$$[E_f] = [L]^{-1}_{0dq} \begin{bmatrix} R_s & 0 & 0 & 0 & 0 & 0 \\ 0 & R_s & \omega L_{ss} & 0 & 0 & \omega M \\ 0 & -\omega L_{ss} & R_s & 0 & -\omega M & 0 \\ 0 & 0 & 0 & R_r & 0 & 0 \\ 0 & 0 & (\omega-\omega_r)M & 0 & R_r & (\omega-\omega_r)L_{rr} \\ 0 & -(\omega-\omega_r)M & 0 & 0 & -(\omega-\omega_r)L_{rr} & R_r \end{bmatrix} \tag{9.6c}$$

and

$$L_{ss} = L_s + L_{sm}$$

$$L_{rr} = L_r + L_{mr}$$

$$M = 3/2\, L_{sr} \tag{9.6d}$$

The zero sequence quantities are zero if the machine is symmetrical and the three-phase voltage is balanced. The zero sequence quantities can be neglected in the analysis.

Neglecting zero sequence quantities in Eq. 9.6c and rearranging the equations gives

$$p[i] = [D]\,[v] + [E]\,[i] \tag{9.7a}$$

where

$$[i] = [i_{qs} \;\; i_{qr} \;\; i_{ds} \;\; i_{dr}]^T \tag{9.7b}$$

$$[v] = [v_{qs} \;\; v_{qr} \;\; v_{ds} \;\; v_{dr}]^T \tag{9.7c}$$

For singly-fed induction machines,

$$v_{qr} = v_{dr} = 0. \tag{9.7d}$$

The [D] and [E] matrices are given in Eq. 9.7e and Eq. 9.7f.

In the steady state, the power input into the machine is given by Eq. 9.8a.

$$p_{in} = v_{qs}i_{qs} + v_{ds}i_{ds} \tag{9.8a}$$

The reactive power input is also given by Eq. 9.8b.

$$q_{in} = v_{ds}i_{qs} - v_{qs}i_{ds} \tag{9.8b}$$

Substituting for v_{qs} and v_{ds} from Eq. 9.3p in Eq. 9.8a gives

$$\begin{aligned} p_{in} &= (-\omega L_{ss}i_{ds} + R_s i_{qs} - \omega M i_{dr})i_{qs} \\ &\quad + R_s i_{ds} + \omega L_{ss}i_{qs} + \omega M i_{qr})i_{ds} \end{aligned} \tag{9.8c}$$

$$= R_s(i_{qs}^2 + i_{ds}^2) + \omega M(i_{ds}i_{qr} - i_{dr}i_{qs}) \tag{9.8d}$$

The first term in Eq. 9.8c is the i^2R power loss in the stator and the second term is the air-gap power transferred between stator and rotor. From the second term, the electromagnetic torque developed by the machine is found to be

$$T_e = M(i_{ds}i_{qr} - i_{dr}i_{qs}) \tag{9.8e}$$

If the machine drives a mechanical load, the torque (T_m) developed by the load would vary with the speed of rotation (ω_r). Eq. 9.9 gives a general description of the

$$[D] = \Delta \begin{bmatrix} -L_{rr} & M & 0 & 0 \\ M & -L_{ss} & 0 & 0 \\ 0 & 0 & -L_{rr} & M \\ 0 & 0 & M & -L_{ss} \end{bmatrix} \qquad (9.7e)$$

$$[E] = \Delta \begin{bmatrix} L_{rr}R_s & -MRr & -\omega L_{ss}L_{rr} + (\omega-\omega_r)M^2 & -\omega_r L_{rr}M \\ -MR_s & L_{ss}Rr & \omega_r L_{ss}M & \omega M^2 - (\omega-\omega_r)L_{ss}L_{rr} \\ \omega L_{ss}L_{rr} - (\omega-\omega_r)M^2 & \omega_r ML_{rr} & L_{rr}R_s & -MRr \\ -\omega_r ML_{ss} & -\omega M^2 + (\omega-\omega_r)L_{ss}L_{rr} & -MR_s & L_{ss}Rr \end{bmatrix}$$

(9.7f)

where

$$\Delta = 1/(M^2 - L_{ss}{}'L_{rr}) \qquad (9.7g)$$

mechanical load torque.

$$T_m = a + b\omega_r + c\omega_r^2 + d\omega_r^\beta \tag{9.9}$$

where

T_m	= torque developed by mechanical load
ω_r	= speed of machine and load
a, b, c, d	= mechanical torque coefficients (some of the coefficients could equal zero)
β	= an exponent

From Newton's law, the inertial dynamics of the rotor and mechanical load are given by Eq. 9.10

$$p\omega_r = (T_e - T_m)/J \tag{9.10}$$

where, J = the total inertia of motor and load

By definition,

$$H = \frac{\text{kinetic energy stored at synchronous speed (kW)}}{\text{base kVA}}$$

$$= W_k/S_b \tag{9.11}$$

where

W_k = stored kinetic energy of the rotor and load

S_b = base kVA of the machine

But the stored kinetic energy is

$$W_k = (1/2)\, J\omega_b^2 \tag{9.12}$$

Therefore,

$$HS_b = W_k = (1/2)\ J\omega_b^2 \tag{9.13}$$

ω_b is the synchronous speed.

$$J = 2HS_b/\omega_b^2 \tag{9.14}$$

Hence,

$$p\omega_r = (T_e - T_m)\ \omega_b^2/2HS_b \tag{9.15}$$

That is,

$$\omega_b\ p(\omega_r)_{pu} = (T_e - T_m)_{pu}\ T_b\ \omega_b^2/2HS_b \tag{9.16}$$

where

T_b = the base torque

$$T_b\omega_b = S_b \tag{9.17}$$

Therefore,

$$\omega_b\ p(\omega_r)_{pu} = (T_e - T_m)_{pu}\ \omega_b/2H \tag{9.18}$$

Hence, in per unit quantities,

$$p\omega_r = (T_e - T_m)/2H \tag{9.19}$$

X. APPENDIX B: CONVERSION OF MACHINE PARAMETERS AND VARIABLES FROM THE MACHINE BASE TO A COMMON SYSTEM BASE

A. Conversion of Machine Impedances

The parameters of machines are usually given in per unit on machine base. In order to group two or more machines into an equivalent, it is necessary to convert these parameters to a common base, usually the system base. Consider an induction machine that is rated for S_k volt-amperes at V_k volts which is connected in a power system. If the base voltage of the power system is V_b and base voltampere of the system is S_b, then the base impedance of the power system is

$$Z_b = V_b^{\,2}/S_b \tag{10.1}$$

and the base impedance of he machine is

$$S_{bk} = (V_k)^2/S_k \tag{10.2}$$

If z_k is a per unit impedance of a circuit in the k-th machine on its own base, z_k may be converted to the system base (Z_b) by

$$(z_k)_{\text{system base}} = z_k \; Z_{bk}/Z_b \tag{10.3}$$

$$= (z_k) \; (V_k/V_b)^2 \; (S_b/S_k) \tag{10.4}$$

B. Conversion of Machine Currents

Let the per unit current drawn by the k-th machine be denoted by i_k. This is usually calculated on machine base. If v is the per unit voltage on the machine, then the actual voltamperes $(VA)_k$ drawn by the k-th machine is

$$(VA)_k = (v\ i_k)_{pu}\ S_k \tag{10.5}$$

On the system base, the per unit apparent power $(VA)_{k,pu}$ drawn by the k-th machine is

$$(VA)_{k,pu} = (VA)_k/S_b = v\ i_k\ (S_k/S_b) \tag{10.6}$$

From Eq. 10.6, the per unit currents drawn by the k-th machine may be expressed on system base by multiplying the per unit current on the machine base by the ratio (S_k/S_b).

C. Conversion of the Mechanical Torque Coefficients

It is assumed that the equation relating the mechanical torque (T_m) to machine speed ω_r is given on the machine voltampere base to be

$$T_m = a + b\omega_r + c\omega_r^2 + d\omega_r^\beta \tag{10.7}$$

If the coefficients a, b, c and d are expressed in per unit on a machine voltampere base of S_k, then the actual power (P_k) developed by the k-th machine is

$$P_k = (T_{mk}\omega_{rk})(S_k)$$
$$= (a\ \omega_{rk} + b\ \omega_{rk}^2 + c\ \omega_{rk}^3 + d\ \omega_{rk}^{\beta+1})\ (S_k) \tag{10.8a}$$

where

T_{mk} = per unit torque developed by the k-th machine given on machine base

ω_{rk} = per unit speed of the k-th machine

Expressed in per unit on system base, the per unit power developed by the k-th machine is

$$P_k/S_b = (T_{mk}\omega_{rk})\ (S_k/S_b)$$
$$= (a\ \omega_{rk} + b\ \omega_{rk}^2 + c\ \omega_{rk}^3 + d\ \omega_{rk}^{\beta+1})\ (S_k/S_b) \tag{10.8b}$$
$$= a(S_k/S_b)\ \omega_{rk} + b(S_k/S_b)\ \omega_{rk}^2 + c(S_k/S_b)\ \omega_{rk}^3 + d(S_k/S_b)\ \omega_{rk}^{\beta+1} \tag{10.8c}$$

From Eq. 10.8c, the torque coefficients a, b, c and d can be converted from the machine base to the system base by multiplying each coefficient by the ratio (S_k/S_b).

D. Conversion of the Inertia Constant

The inertia of the rotor and the mechanical load it drives is usually given as an inertia constant H which is expressed on the machine base. This inertia constant is proportional to the stored kinetic energy (Wk) at synchronous speed. If H_k is the inertia constant of the k-th machine and its voltampere rating is S_k, then the stored kinetic energy is

$$Wk = H_k \, S_k \qquad \text{(joules)} \tag{10.9}$$

On the system base (S_b), the stored kinetic energy may be written as

$$Wk = H'_k \, S_b \qquad \text{(joules)} \tag{10.10}$$

where

H'_k = the inertia constant expressed on system base.

From Eqs. 10.9 and 10.10,

$$H'_k = H_k \, (S_k/S_b) \tag{10.11}$$

XI. APPENDIX C: FINDING THE INITIAL CONDITIONS OF INDUCTION MACHINE VARIABLES

A. Iterative Solution of the Steady State Equivalent Circuit

If the parameters of an induction machine, the mechanical power output (or the torque description of the mechanical load) and the voltage on the machine are known, the initial conditions of the current, electrical power input, speed and torque can be calculated using the steady state equivalent circuit. The equivalent circuit of an induction machine is shown in Fig. 11.1.

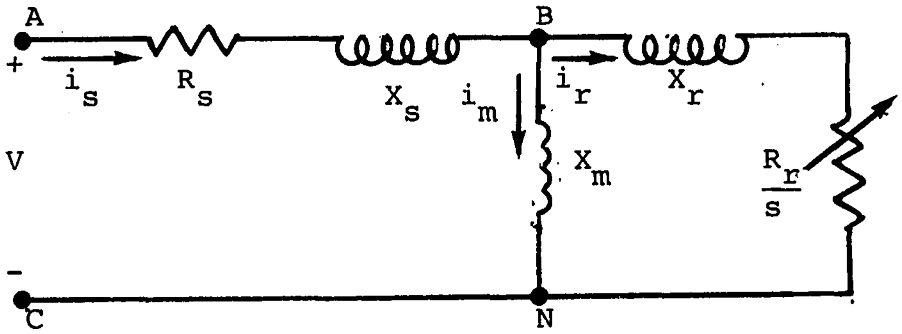


Fig. 11.1. Steady state equivalent circuit of an induction machine

We can find Thevenin's equivalent voltage (V_{th}) and impedance (Z_{th}) of the source as seen at the magnetizing branch (BN in Fig. 11.2). These are given in Eq. 11.1 and Eq. 11.2.

$$V_{th} = jX_m/(R_s = j(X_s + X_m))\ V \qquad (11.1)$$

$$Z_{th} = jX_m\,(R_s + jX_s)/(R_s + j(X_m + X_s)) = R1 + jX1 \tag{11.2}$$

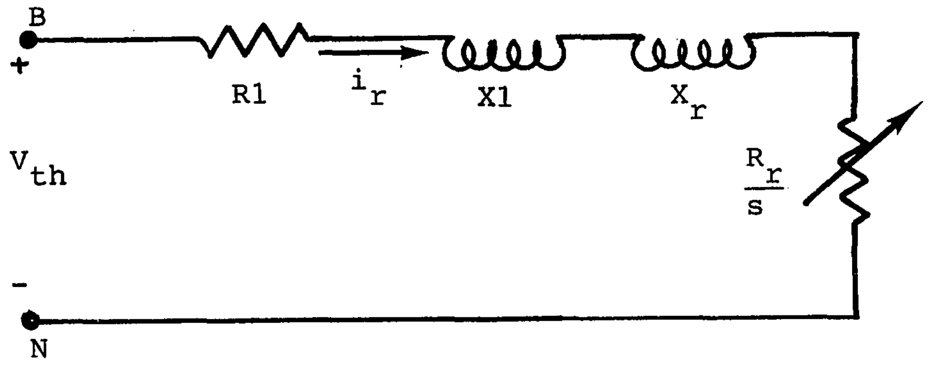


Fig. 11.2. Reduced equivalent circuit of Fig. 11.1

The rotor current i_r is given by

$$i_r = V_{th}/((R1 + R_r/s) + j(X1 + X_r)) \tag{11.3}$$

The total power delivered to the rotor is $i_r^2\ R_r/s$ and rotor resistive loss is $i_r^2\ R_r$. The mechanical power output is

$$P_m = i_r^2\ R_r\ (1 - s)/s \tag{11.4}$$

The speed of the rotor is (1 - s) and hence the torque (T_m) developed is

$$T_m = P_m/(1 - s) = i_r^2\ (R_r/s) \tag{11.5a}$$

$$= V_{th}^2\ (R_r/s)/((R1 + R_r/s)^2 + (X1 + X_r)^2) \tag{11.5b}$$

Therefore,

$$P_m[(R1 + R_r/s)^2 + (X1 + X_r)^2] = (R_r/s)\ (1 - s)\ V_{th}^2 \tag{11.6}$$

This reduces to

$$(R_r/s)^2 + [2R1 - (1-s)\, V_{th}^2/P_m]\, (R_r/s) + (X1 + X_r)^2 = 0 \quad (11.7)$$

or

$$(R_r/s)^2 + [2R1 - V_{th}^2/P_m]\, (R_r/s) + (X1 + X_r)^2 + V_{th}^2\, R_r/P_m + R1^2 = 0 \quad (11.8)$$

Equation 11.8 is in the form

$$y^2 + fy + g = 0$$

where

$$y = R_r/s \quad (11.9)$$

$$f = 2R1 - V_{th}^2/P_m \quad (11.10)$$

$$g = (X1 + X_r)^2 + V_{th}^2/P_m \quad (11.11)$$

At any given voltage V, V_{th}, f and g can be calculated. The solution of the quadratic equation above is given by

$$y = (-f \pm \sqrt{f^2 - 4g})/2 \quad (11.12)$$

Since R_r/s is positive for motors,

$$y = (-f + \sqrt{f^2 - 4g})/2 \quad (11.13)$$

Given the value of the mechanical output power P_m and voltage V, an iteration can be carried out with an initial guess of the slip just above 0 pu (e.g., s = 0.02). The value of the quantity $y = R_r/s$ may then be calculated. A new value for the slip can then be determined ($s = R_r/y$) and the calculations continued until convergence.

If the torque description $T_m = a + b\omega_r + b\omega_r^2 + d\omega_r^\beta$ instead of P_m were known, this will just introduce one more step in the iterative process. From the guessed value of slip s, the rotor speed ($\omega_r = 1 - s$) and the torque may be computed. The mechanical power output, $P_m = T_m\omega_r$, may then be calculated and the iteration process continued.

After the slip is determined, Eq. 11.3 may be solved to obtain the value of the rotor current i_r. The stator current is given by

$$i_s = i_r + i_m \tag{11.14}$$

where i_m is the magnetizing current and it is given by

$$i_m = i_r(R_r/s + jX_r)/jX_m \tag{11.15}$$

Therefore,

$$i_s = i_r[1 + (R_r/s + jX_r)/jX_m] \tag{11.16}$$

If the voltage phasor is taken as a reference, then the phase angle (αs) of the stator current is the power factor angle. Hence,

$$i_s = |i_s| \angle{-\alpha s} \tag{11.17a}$$

Similarly,

$$i_r = |i_r| \angle{-\alpha r} \tag{11.17b}$$

where,

$|i_s|$ and $|i_r|$ are the magnitudes of the phasors i_s, and i_r

αr is the power factor angle of the rotor circuit

B. Transformation of Machine Variables into 0-d-q Axes

For balanced three-phase voltages, the d-axis and the q-axis components of the stator voltage are given by

$$v_{ds} = v \cos(\omega t) \qquad (11.18a)$$

$$v_{qs} = v \sin(\omega t) \qquad (11.18b)$$

where

v = per unit value of the voltage on the stator

ωt = angle by which the d-axis leads the reference phase voltage (the a-phase of the a-b-c system)

ω = $2\pi f$

t = time in seconds

f = frequency in hertz

If the stator phase current is $|i_s|\underline{/-\alpha s}$ and the rotor current is $|i_r|\underline{/-\alpha r}$, then, in the 0-d-q axes,

$$i_{ds} = |i_s| \cos(\omega t - \alpha s) \qquad (11.19a)$$

$$i_{qs} = -|i_s| \sin(\omega t - \alpha s) \qquad (11.19b)$$

$$i_{dr} = |i_r| \cos(\omega t - \alpha r) \qquad (11.19c)$$

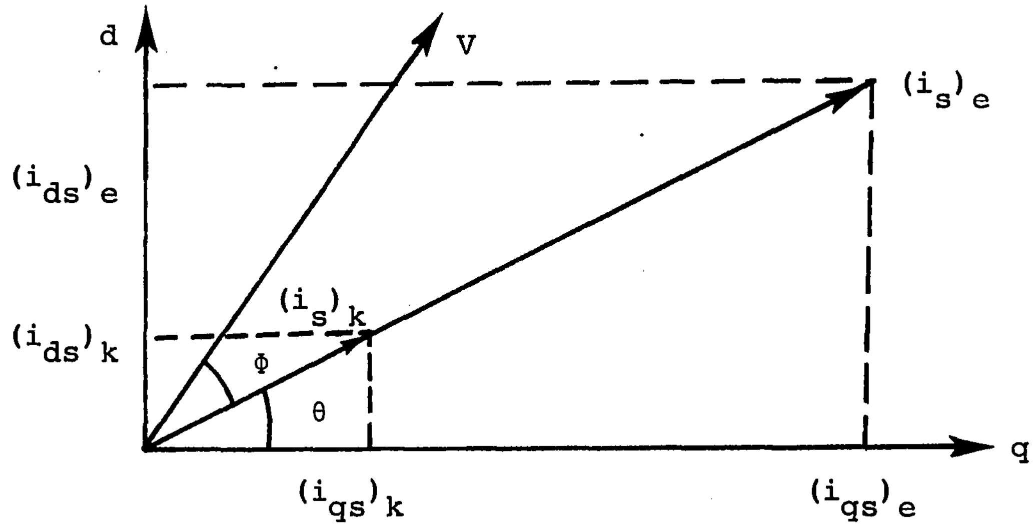
$$i_{qr} = -|i_r| \sin(\omega t - \alpha r) \qquad (11.19d)$$

These are the steady state initial conditions required for modeling the induction machine.

XII. APPENDIX D: DERIVATION OF APPROXIMATE WEIGHTING FACTORS

A. Proof That for Equal Power Factors, $(i_{qs})_k/(i_{qs})_e = (i_{ds})_k/(i_{ds})_e = (kVA)_k/(kVA)_e$

The phasor diagram shows the relationship between the voltage and currents of a number of induction machines connected to the same bus.



The voltage V is common to all machines. If the power factors of all machines are equal, the currents of the individual machines will be in phase with each other and with the current of the equivalent machine, which is the sum of the currents to the individual machines.

From the phasor diagram,

$$(i_{ds})_k = (i_{qs})_k \tan \theta \tag{12.1}$$

$$(i_{ds})_e = (i_{qs})_e \tan\theta \tag{12.2}$$

Taking the q-axis as reference, kVAs of the k-th machine and the equivalent machine are

$$(kVA)_k = Vi_k = V[(i_{qs})_k + j(i_{ds})_k]\underline{/\phi + \theta} \tag{12.3}$$

$$(kVA)_e = Vi_e = V[(i_{qs})_e + j(i_{ds})_e]\underline{/\phi + \theta} \tag{12.4}$$

Therefore,

$$\frac{(kVA)_k}{(kVA)_e} = \frac{V[(i_{qs})_k+j(i_{ds})_k]}{V[(i_{qs})_e+j(i_{ds})_e]} = \frac{(i_{qs})_k}{(i_{qs})_e}\frac{(1+j\tan\theta)}{(1+j\tan\theta)} = \frac{(i_{qs})_k}{(i_{qs})_e} \tag{12.5}$$

Similarly,

$$\frac{(kVA)_k}{(kVA)_e} = \frac{V[(i_{qs})_k+j(i_{ds})_k]}{V[(i_{qs})_e+j(i_{ds})_e]} = \frac{(i_{ds})_k}{(i_{ds})_e}\frac{1/\tan\theta+j}{1/\tan\theta+j} = \frac{(i_{ds})_k}{(i_{ds})_e} \tag{12.6}$$

B. Percentage Error in the Equal Power Factor Assumption

If all machines operate at equal power factors, the total voltamperes drawn by all machines is just the arithmetic sum of that drawn by the individual machines. Seen from the common bus, the voltampere of each machine is proportional to its line current. Figure 12.1 illustrates the addition of two voltamperes and the errors involved in using an approximation. From the figure,

$$\bar{I}_1 = I_1\underline{/-\theta_1} \tag{12.7}$$

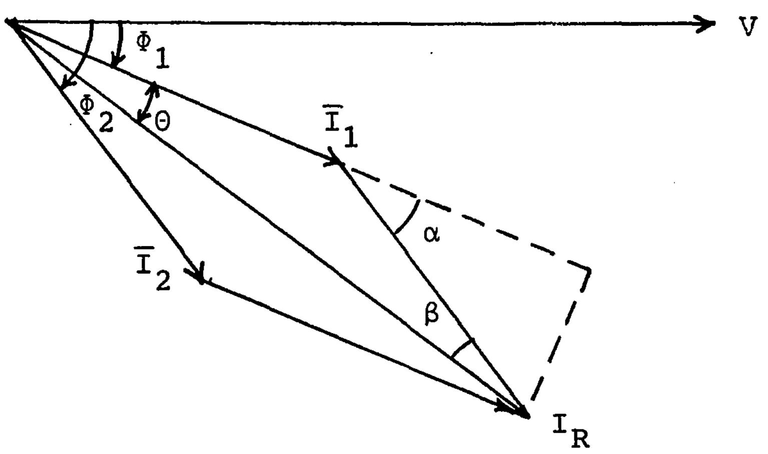


Fig. 12.1. Addition of two current phasors

$$\overline{I}_2 = I_2\underline{/-\Phi_2} \tag{12.8}$$

$$\Phi_2 - \Phi_1 = \Phi + \beta + \alpha \tag{12.9}$$

The resultant current

$$\overline{I}_R = \overline{I} + \overline{I}_2 = (I_1 \cos\theta + I_2 \cos\beta)\underline{/-\Phi_1 - \theta} \tag{12.10}$$

But by assuming equal power factors, the approximate resultant current is

$$\tilde{I}_R = I_1 + I_2 \tag{12.11}$$

The correct total voltamperes is given by Eq. 12.12,

$$VA = \overline{V}I_R \tag{12.12}$$

the magnitude of which is

$$|V\overline{I}_R| = V(I_1 \cos\theta + I_2 \cos\beta) \tag{12.13}$$

The approximate total voltamperes is given by Eq. 12.14,

$$VA = VI_R = V(\tilde{I}_1 + I_2) \tag{12.14}$$

The per unit error in using the approximate VA is

$$\frac{VI_R - |V\bar{I}_R|}{|V\bar{I}_R|} = \frac{(I_1 + I_2) - (I_1 \cos\theta + I_2 \cos\beta)}{I_1 \cos\theta + I_2 \cos\beta} \quad (12.15)$$

$$= \frac{I_1 + I_2}{I_1 \cos\theta + I_2 \cos\beta} - 1 \quad (12.16)$$

For practical motors, the full load operating power factors range from 0.65 for small motors to 0.91 for large motors [39]. The corresponding angles for Φ_2 and Φ_1 are arccosine (0.65) or 49.5°, and arccosine (0.91) or 24.5°. These give the maximum phase angle between the currents drawn by any two induction motors operating within the above range of power factors as 25°.

Given the magnitudes of $\bar{I}_1$ and $\bar{I}_2$ and the angular spread ($\alpha = \beta + \theta$) between them, the values of θ can be computed as follows.

$$\theta = \arctan\left[\frac{I_2 \sin\alpha}{I_2 \cos\ \ + I_1}\right] \quad (12.17)$$

$$\beta = \alpha - \theta \quad (12.18)$$

The per unit error of Eq. 12.16 can then be computed.

Numerical calculations have been done for various values of I_1 and I_2 and α. As shown below, the maximum error occurs when I_1 equals I_2 and α is at its maximum possible values (25° in this case). This percentage error is 2.4%. Table 2.1 shows the results for various values of I_1/I_2 and α.

The conditions for maximum error can be derived as follows. Let error be

$$\frac{\Delta VA}{VA} = \frac{I_1 + I_2}{I_1 \cos\theta + I_2 \cos\beta} - 1 \tag{12.19}$$

$$\frac{\partial}{\partial I_1}\left(\frac{\Delta VA}{VA}\right) = \frac{I_1 \cos\theta + I_2 \cos\beta - (I_1+I_2) \cos\beta}{(I_1 \cos\theta + I_2 \cos\beta)^2} \tag{12.20}$$

$$= \frac{I_2 (\cos\beta - \cos\theta)}{(I_1 \cos\theta + I_2 \cos\beta)^2} \tag{12.21}$$

Table 12.1. Per unit errors in the equal power factor assumption

α	I_1/I_2	1	2	5	10	20
25°		0.024	0.021	0.013	0.008	0.004
22°		0.019	0.016	0.010	0.006	0.003
19°		0.014	0.012	0.008	0.005	0.002
16°		0.010	0.009	0.005	0.003	0.002
13°		0.006	0.006	0.004	0.002	0.001
10°		0.004	0.003	0.002	0.001	0.001
7°		0.002	0.002	0.001	0.001	0.000

For maximum or minimum,

$$\frac{\partial}{\partial I_1}\left(\frac{\Delta VA}{VA}\right) = 0 \tag{12.22}$$

Therefore,

$$\beta = \theta = \frac{\alpha}{2}; \quad (\beta = \alpha - \theta) \tag{12.23}$$

Also,

$$\frac{\partial}{\partial\theta}\left(\frac{\Delta VA}{VA}\right) = \frac{-I_1 \sin\theta + I_2 \sin(\alpha-\theta)}{(I_1 \cos\theta + I_2 \cos(\alpha-\theta))^2} \tag{12.24}$$

For maximum or minimum,

$$\frac{\partial}{\partial\theta}\left(\frac{\Delta VA}{VA}\right) = 0 \tag{12.25}$$

Therefore,

$$I_1 \sin\theta = I_2 \sin(\alpha-\theta) \tag{12.26}$$

Using the condition ($\theta = \alpha/2$) of Eq. 12.23 in Eq. 12.26,

$$I_1 = I_2$$

for minimum or maximum error.

The second derivative of the per unit error function is

$$\frac{\partial^2}{\partial\theta^2}\left(\frac{\Delta VA}{VA}\right) = \frac{G\ (G^2 + H^2)}{G^4} < 0 \tag{12.27}$$

Hence, maximum where

$$G = I_1 \cos\theta + I_2 \cos(\alpha-\theta) > 0 \text{ for } 0 < \theta < \alpha < 25° \tag{12.28}$$

and

$$H = -I_1 \sin\theta + I_2 \sin(\alpha-\theta) \tag{12.29}$$

These analyses show that the maximum error in the equal power factor assumption occurs when two currents of equal magnitudes at their widest phase separation are assumed to be in phase.